1. Of enjoyment

Interdiction is another figure in the play between the saying and the said. For us, this interdiction is structure, and it is in order to explicate this structure when we draw traces out of the issue that is posed to the subject. The subject is seen obstructed by the presence of a body, his own or the other’s. He does not know how to make do with a body: this is the question of enjoyment (jouissance), the question of the use of the body.

a – Written presentation

The appearance of the term enjoyment is vouched for in analytic discourse with two of Lacan’s writings from the year 1958.

In the first: “The Signification of the Phallus,” words pronounced in Munich on the 9th of May in 1958,¹ he makes reference to the masturbatory enjoyment of the phallic phase (E., p. 693) that will be converted into phallic enjoyment, JΦ.

In the second: “The Youth of Gide or the Letter and Desire,” it appeared in April of the same year² in relation with Freud's Ego ideal in which he mentions the enjoyment of a desire to define how this instance is formed through the adoption of the image of an Other. This Ego ideal is painted on a mask whose function will occupy our attention in chapter VII. This is made “with the repression of a desire of the subject” from which the Other “has enjoyment together with rights and means” (E., p. 762, “p”). With this we recall already what in the discourse of psychoanalysis will be converted into the enjoyment of the Other, JA.

We insist on the juridical coordination of enjoyment, with what we end up bumping up against concerning rights, supposing that this seems essential. This coordination affords us the opportunity of linking the Ego ideal with kinship. We find this meaning of enjoyment in a writing of Lacan dated

¹ We are not disposed to the date of writing, which we consider contemporaneous because this text was not published before the appearance of the Écrits.
² In number 131 of the journal Critique.
In a way contrary to the analogies of those who chuck around abuses, Lacan explains in which sense a technique, like the cultivation of the soil, can be qualified from the metaphor of coitus. In the measure by which a technique is an object of interdiction, interdiction is recast for this example in the enjoyment of the soil and produced in the roots of the rules of alliance and kinship, whose combinatorial function we have situated here, the technique that replaces the first one is rendered symbolic of a sexual satisfaction. The technique is interdicted in reason from a block, a knot, an impossibility in the combinatorial network while this rule of the things' enjoyment is placed under the weight of repression.

The 1972 presentation of the concept of enjoyment, in the “Aun” seminar, which takes up and extends the seminar about “the Ethics of psychoanalysis,” insists upon this juridical dimension of enjoyment, defining it through usufruct. It always functions according to the same laws that rule the bequest of the use of some goods, of its enjoyment or of some titles entrusted to the subject, on the condition of not abusing them, which is precisely to transmit them (S. XX). The Freudian instances are made, in repression, from the singularities and accidents of this network.

enjoyment is the use of the body within its cultural and juridical coordinates, but it is also the place “whose lack would make the universe empty,” even though it would make “Being itself languish.” enjoyment responds to what I am. But instead of telling myself that I am a piece of furniture, it designates a site in a different manner: “I am in the place from where it is shouted that “the universe is a defect in the purity of non-being” (E., p. 819, “v”).

We place these elements, that are hardly cheering (rejouissants), to clarify this third coordinate linked to structure. We would see now where this concept and its questions comes from in the work of Freud.

a’ - Where enjoyment is found in the Freudian construction

The question of enjoyment is posed since “A sexual theory,” when Freud is occupied with sexual tension (Spann) and the mechanism of priminary pleasure. The mechanism of sexual excitation seems paradoxical and even contradictory if we consider pleasure as a diminution of tension, as Freud maintains this firmly in this precise place of his text ([1 e], p. 114, [1 e’], p. 11). As a matter of fact, we do not understand how the diminution and prolongation of tension (pleasure) can be looked for in the same gesture, including the augmentation of it with views to a much greater pleasure ([1 e], p. 115; [1 e’], p. 13).

“On one side, this stimulation is already tied to pleasure; on the other, as a consequence this has an elevation of the state of sexual excitation or an awakening of it there where it still lacks.” ([1 e’], p. 13)

Additionally, just further down:

“This molestation already provokes a pleasurable sensation but simultaneously it is appropriated like no other to awaken a sexual excitation that demands a gain in pleasure.” ([1 e’], p. 15)
“Then the effect is the same: on one side, there is pleasurable sensation that is quickly seen strengthened by the pleasure deriving from the preparatory modifications, on the other, a new elevation of sexual tension that is very quickly converted into the more notorious displeasure if it does not get the opportunity to provoke another pleasure.” (Ibid.)

The “problem whose solution would be as difficult as important for the conception of the sexual processes” ([1 e], p. 114; [1 e'], p. 9), we read and consider to be of the structure of the signifying involution.

“The problem lies, justly, in the way in which the experienced pleasure generates the necessity of a greater pleasure.” ([1 e'], p. 15; [1 e], p. 116.)

This is the problem of surplus enjoyment that, as the translators we cite point out, we see arising out of Freud's pen when he speaks of this “gain in pleasure.”

The point is not to know if this principle should be named pleasure, displeasure, or pleasure-displeasure. In this case this would be nothing more than a discussion of words which would not render an account of the paradox of enjoyment.

In the paragraph added in 1920 to his third essay on the theory of sexuality, Freud completes his argumentation by means of the libido theory. The paradox of enjoyment, the enigma of human sexuality, is converted into the difference and identity of two libidos that he identifies as ego libido and sexual-object libido. We return to clearly find the main question of structure which, in our Presentation, we were saying was the moment of Freud.

{56}

For us, this paradox is structure: the structure of language, of the signifier, castration—here the paradox of enjoyment. It can only be approached by means of a metaphor, what Lacan calls the imaginary function of the phallus, to say what in the moment in which it is discovered is not a simple mystification, which means that not only can a veil be thrown over it, but that it is elided. This structure of the interdiction is the structure of shame in which the devil of the Eidos (the devil of shame) jumps to the scene in the instant of the deviling of the phallus (E., pp. 555 and 692; “o” and “q”).

In order to be construed in logic, this structure requires a topology. It is realized thanks to an interior operator, the function of the Other that modifies classic logic before being elided to give access to the field of fictions. The Other is crossed out by its elision and shows in which sense it does not cease to stop writing itself, but exists like how imaginary numbers exist for real numbers. This is how we refer ourselves to this structure: an interdiction falling back on the Other. A structure that is lack in the Other and of its enjoyment, a thing which one must render an account of.

It is topological structure whose reliefs we develop through topological manifolds: graphs, surfaces, knots. This is the moment of the experience without which it does not spout any symptomatic (phobia) or structural (Penisneid) consequences (E., p. 693, “q”).

---

3 Nons, fascicle #0
“Of course, its future depends on the law introduced by the father in this sequence” (E., p. 694, “q”). The law, the combinatory of the signifier, is situated in Lacan's schemas between I and A, characterizing Freud's perception-signs, in opposition to the pure, raw perceptions of physiology that do not experience this sequential cutting. But the Imaginary depends on the Symbolic, and violence itself rests on a pact. Therefore, the resurgence of rebellion depends on a flaw of this structure, and from a flaw in the articulation of it, the maniacal return to sadness. A return that is mortal (Television, p. 39).

a” - Structural Presentation

If we do not lose sight of the trace of the schemas on the surfaces, we can read and show in our conclusion in which aspects the signifying involution accounts for the manner in which what is first is knotted with the secondary. This clarifies the composition of the superego with Freud's Ego ideal, and of the perception-signs with the preconscious, and in which sense enjoyment can be understood as {57} heard-unjoy-sense (j'ouis sens), the essence of the word, a bit of discourse that constitutes the voice of conscience. The paradox of enjoyment, whose soul is falsification, as we said earlier, results in the necessity of guilt in neurosis. This is maintained while the anxiety that separates satisfaction from desire is not purified. This purification is writing.

It makes time lack for writing this paradox, always the same one, with some clarity, which is formulated one more time in another part in relation with the father, in this other figure:

“not only does the murder of the father not open the way through enjoyment, [the father] whose presence was supposedly prohibiting it, but [the murder] reinforces his interdiction.” (S. VII, p. 207; “214.”)

The father is only the supposed mediator of the interdiction of enjoyment. With respect to him we can distinguish already the imaginary rivalry, on one side, from the law whose authority he assumes, accompanied, on the other side, by the elision that he is correlative to. The notion of murder mixes this up. From this point of view we can write the equation that composes the Imaginary with the Symbolic.

“It's all there, and really there, even in the deed as in the explication, the phallus.” (S. VII, p. 207; “214”)

This equation is the phantasm that we cannot pretend to get out of, but rather, to render an account of; thanks to mathematics, we make logical matheme from this same impossibility, from this phallus.

This logical solution, to no longer be stuck in the Imaginary, confirms for us that:

“The obstacle abolished under the form of the murder, not for this [is] enjoyment less interdicted; more even, the interdiction is reinforced.” (S. VII, p. 207; “214.”)

But at the same time we transmit the profile of this difficulty and its dissolution.

Structure goes through the body or through thought. Thus, as a result of this interdiction, from the juices that constitute it—the Other, the inversion and the veil—, enjoyment is divided necessarily as phallic enjoyment and enjoyment of the Other. This two enjoyments are not, respectively, substance of
extension (the body in hysteria) and substance of thought (the soul in obsessive neurosis). They induce another substance, unjoyed substance (the object of phobia), the position of inflection in the symptomatology of the neuroses. Certain masks display this substance when they extend themselves into a territory, an extension from the libido when language reigns in it, therefore coming from but distinct from ethology.

The relapsed interdiction in enjoyment conducts necessarily the break from a first, we would say, phallic, enjoyment. This is the manner in which metaphor achieves the breaking, leaving to hear what is not said and implying the search for another thing, we would say, the enjoyment of the Other, the effect of ravishment. Satisfaction, always disappointed in the subject, carries him away to suspect another enjoyment to reach for. But to leave things this way is insufficient: this is a weak preference of the interdiction which he who does not structure into a good logic remains adherent to. This other enjoyment, if it exists, would not be this; what would be lacking is what could not make lack (S. XX, pp. 54-57; “73-76”). This makes deception absolute and resolve it in a redoubled manner.

But instead of contenting ourselves with paraphrasing the elegant presentation which Lacan gave, we must write the logic of the profile of this fall in an axiomatizable way, to modify Boole's algebra and Kant's reason, when this construction shows us in which sense it entails a consummation. We translate this same impossibility into mathematics. By not renouncing to write this structure, we demonstrate, by means of the realization of its same break, that by which it does not write itself. This structure of enjoyment is that of language, in accordance with there being no metalanguage, on the condition of construing this new negation and running through its temporality.

The veil ratifies the trivialization of this other logic and permits us the study of the structural rules that condition this banalization. We see ourselves obliged to reason about false enunciations and empty sets. This can seem a logical lack, because the academic world reputes the false as defective. This lack averts the rationalizations of the subject, and through an inversion this is attributed to and converted into lack's fault. Supposing that the subject reasons with clarity on the false and it is false that he would not know, in the textual manner, that although he would not have and never was going to have access to the two sexes but rather, to this phallic enjoyment (masturbatory or coital), his enjoyment depends on an other, and other enjoyment that he will not have because it could not be so. But it is false that he would know that he cannot and that he will not dare to allude to it, in its logical articulation: so great is the intimidation that the fear of being fallible provokes, by being culpable of a logical fault.

It is false then that he would not know neither and at the same time that to be resigned to phallic enjoyment renders a bad service to the penile organ, in consideration of this other enjoyment, and which commits him that way to a lack of relish.

{59}

We comprehend in such a way that Freud manifested reservations in the discussion related to onanism, even with respect to the liberal thesis as a repressive position. Supposing that the problem does not reside in this, in view of the fact that, in this domain, it is without a doubt from the existence of another logic.

---

4 Nons, fascicle #0
At the end of the second chapter of Lacan’s teaching, the signifying involution forms us in this dialectic; these are our exercises of exchange between bilateral and unilateral surfaces. This way the topology of the subject is projected, and this intermediary stage gives nothing more than a weak idea of the reason for its break (“La méprise du sujet...”; “Raison d’un échec”). Whereas, it already affords the necessary means for the investigation into the space of the knot. Falling back from the phallic veil makes the following turn more difficult, as Dr. Lacan explains it when he undertakes his new proof (“On the lecture of December 17,” p. 98). In this transcription, he tries to consider its existence from the insistence of the Real as (an imposible) hole. The insistence of the hole passes then to the Symbolic.

Even those who are entangled in the theory of intrinsic topological surfaces have guaranteed phallicism, given that it supposes another unattainable theory. This incites us to pass to this final stage because it is false that it would be different from what preceded it, and consider to be false that it would be the same. Taken much further uniquely by structure, we do not assume, in the multiplicity of knots, anything else but the position of an encircling more precise than this other substance that there is none of, qualified as phobia, between fabric and cut. It is to this substance that the subject of narcosis is hooked to (doped, as they are called) with the pharmacist or dealers, the diodes and the welder. They transform into this psi-thing (psychosis) those irresponsible ones that flirt with unemployment to intimidate and supplicate the employed, and who basically boil down to little actionists. The drama linked to structure is knotted because it is not less foolish to want to assume the defense of some and abolish all other tension, being this way about learning how to exercise it.

With our covering surfaces (spanning surfaces)\footnote{Parenthesis in English in the original, so I will use Vappereau's english term, spanning surface, to designate the term that would be literally translated as surface of tension, now matching this topological surface of tension with Freud's German term, Spann. (-Marc's note)} we take up again the demonstration that there is no enjoyment of the Other. Such can be the situation of psychoanalysis in 1986 for those who support us.

\{60\}

This search is not infinite, and it is not ideal; it is simplicity, because the knot delimits the ultimate stage of Lacan's teaching, from what produces a topological overturning that erases its history.

2. \textbf{From the swarm to fabric}

\textit{a}$_{1}$ – \textbf{Construction of the tension surface}

We present here a sufficiently suprising use of the calculus of the knot's fundamental group, just as we effectuate it in the fields of the knot's ex-sistence.\footnote{In French, there is a play on “chose psy” and “psychose.” (-Marc's note)} This use leads us to continue on to a greater simplicity, a more reduced calculus, even more pure. The knot's fundamental group, in our presentation, offers us a discrete homology, also in Stokes's theorem algebra. Dr. Lacan relates the Freudian drift (\textit{Trieb}) to this theorem of differential and integral calculus (E., p. 847; “w”). The body is implicated in this drift by the intermediary of the hole that lies on an edge. The subject can check this drift, in each attempt at translation, generating a tension that is maintained until he finds the right word.
Dr. Lacan maintains this transposition from two rules of structure. The first demands that the drive, as the calculus of flux in the mentioned theorem, is produced in a zone relying on an edge. The second is that its effect on the zone is resumed in the calculus along the edge. As Freud used to put it, we are without a doubt there in the more mythological register of analytic discourse, not to say on the level of the drives. For us this reference finds its reason in the work of translation, instead of any physical model. Our version links the theorem with the commentary given by Dr. Lacan in his Seminar XI (lecture of May 13, 1964), in which the most pure presentation of the drive circuit is located on a trajectory that travels around the space encircling an edge. To calculate in the signifying swarm, is it not to live the drive in a practice of translation? To advance in this field, one must read with clarity that this translation acts upon two registers that should be traversed in order to be resolved. This proof turns around more purely for the subject in the case of passing through them various times. This frees the word.

With this end, in our apprenticeship, the point is to obtain, through a quotient of the group, that is to say, a simplification of the calculus, a coloring of the zones that define a surface exactly whose knot under consideration is the unique edge. This is where we see that the tension \((\text{Spann})\) depends on the edge knot, and it is where we can place, with new investments on this fabric, the question of consistency, that of the paradox of enjoyment, the tensional structure of language and its resolution by means of a pertinent and interpretative cut. Tension attains its establishment from the place of this cut, and in this sense its construction appears the analysand's concern.

We call spanning surface this fabric deduced from our calculations, as we will now demonstrate. What is important is a transversal cut in the field of the knot's existence, which is already a projection of structure into historicity. To continue, this precipitation will allow us to orient ourselves in the trickling of the knot following its rupture.

Consider simple examples. In the case of the Borromean knot, the calculus gives us, from three generators, the demarcation of the following zones.

\[
\begin{align*}
\alpha &= xyx^{-1}zx \\
\alpha &= yzy^{-1}xy \\
\alpha &= zxz^{-1}yz
\end{align*}
\]

\[\text{Fig. 1}\]

(This example is treated explicitly in our fascicles of results #1, pp. 101-104.)

The three expressions of the zone are annotated to correspond clearly to the definition of this knot's group, because by another aspect we know that it is construed as a free group from the three generators and the two relations.

These two relations are: \(xyx^{-1}zx = yzy^{-1}xy = zxz^{-1}yz\).

---

6 \textit{Essaim}, pp. 48 and 124
To take the quotient, that is to say, to combine certain relations, we obtain the demarcations that define a surface whose edge is a Borromean knot, the same way as Kashmir (cashmere) is the distant mark of an ancient shore.

\[
x = y = z \\
x^2 = 1
\]

Fig. 2

For the complementary relations \( x = y = z \) and \( x^2 = 1 \), we obtain the surface that corresponds to the coloring of the zones annotated as \( x \). This spanning surface is unilateral, and for this reason we color it like a monochrome scrap of fabric.

\[
x = \begin{array}{c}
\end{array}
\]

Fig. 3

When the spanning surfaces are bilateral—we will be occupied later on with the number of faces (see p. 65)—it is known in mathematics as the denomination of Seifert surfaces (named after the mathematician, H. Seifert).

\( a_2 \) – Semi-twists and folds

We formulate the definition of a type of trace found in our drawings that is the most important one after the edge trace. This is about the fold trace.

On the crossings of the knot\(^ 8 \) corresponds semi-twists of ribbon (of spanning surface). Through our calculus we can make folds appear in lieu of the semi-twists.

**Definition of a fold:**

---

\(^7\) *Essaim*, pp. 48 and 124

\(^8\) *Essaim*, pp. 81-85
At a crossing, when the quotient of the group gives 1 value to two zones bordered by the vertex, zones adjacent to the crossing, we can add to it a trace through it that crosses a neutral zone (1 value) and replaces 1 with \( m^{-1}m \) (composed of the inverse of a word, \( m^{-1} \), and the word itself, \( m \)). In this case the two-toned coloring clearly corresponds to the fold of a fragment of bilateral fabric, with the convention from which we assign a color to each letter and then a zone wearing the color of the first letter of the word that names it. (The case of unilateral fabrics does not constitute a problem here, given that \( m=m^{-1} \), simply, by which the two faces are the same color.) We annotate as \( mx \) a word that starts with the expression \( m \), and \( m^{-1}x \) one that begins with the expression \( m^{-1} \).

Note that the letters following the words in question give the different depths of fabrics that can be placed underneath of each zone, in its order of piling up.

The edge portion covered by the first layer of fabric that suspends the gaze, is now stitched. In the case of the fold that we isolate:

\[ mx = \]
\[ m^{-1}x = \]

---

9 *Essaim*, pp. 81 and 83
We see two examples.
These different conventions give rapid results in the case of a ribbon whose edge is a link.

The main point is a ribbon of fabric, like a scrap of jute, shows two folds of the same direction (*sens*).

In accordance with this principle, we can mark the folds of the spanning surface of the Borromean knot that we sketched before (figure 3).
Etlin's translation of *Fabric* - 8/11/9 – circulation is prohibited without my permission - (pp. 51-76)

Fig. 7

Which gives the following coloring, x =

Fig. 8

If we accentuate the deformation of the folds in a continuous manner, changing nothing—neither in the calculus nor in the coloring—then we get this surface:
As a conclusion, we will say that the spanning surface without folds of figure 3 is correct; corresponding to the surface with folds of figure 9.

By means of these calculations and coloring we obtain presentations of knotted fabrics. We find ourselves in a topology extrinsic to the fabrics, as we were explaining in the Introduction (p. 38). This does not oblige us to forget what can be known about the intrinsic topology of a fabric. On the contrary, it is the intrinsic which forgets the extrinsic and not the other way around. In this way, the theory, henceforth a classic one, of intrinsic topological surfaces, to which this work is dedicated, continues to be a reliable practice for us (see chapter II, p. 77). The characteristic properties recognized are, for some, deducible from our calculus, far from the roughness of strong calico. Now one must study these properties in the case of the knots' spanning surfaces.

3. **Intrinsic properties characteristic of a spanning surface**

a1 – *The number of faces*

It is said that a fabric is bilateral when we can distinguish two faces in it (as on a disc).

It is said that a fabric is unilateral when it does not have more than one face only (like the M\øbius strip).

In this case we speak of face and discard the term side, but we conserve as common usage the terms bilateral and unilateral to speak of fabrics.

The number of faces is an extrinsic characteristic of the spanning surface, and it is not defined in the intrinsic topology of fabric (see chapter III, p. 106). In this, the number of faces has a corresponding invariant: orientability.

These two notions are correlative. An orientation of a fabric is defined through a point of reference submerged in it: it is an intrinsic characteristic. The faces are defined by means of vectors perpendicular to the surface. This characteristic appeals to the surrounding space: it is extrinsic. The correlation between these two ways of expressing an identical characteristic is significant, and seems to
be playing on the problem of dimension that is an issue here.
One must make appear this characteristic through two coefficients of the knot's fundamental
group. Our calculus is in fact extrinsic,\textsuperscript{10} it gives the number of faces.
To display this, consider the trefoil knot in its two dual presentations, one with respect to the
other.\textsuperscript{11}

\begin{center}
Fig. 10
\end{center}

First presentation

(0) Basic calculus. The calculus of the fundamental group, for the first presentation, gives us, from two
generators, the following demarcation of the zones,\textsuperscript{12}

\begin{center}
Fig. 11
\end{center}

with the relation \(xy^{-1}x^{-1}yx^{-1}y^{-1} = 1\)

(1) Distinction of wholes and empties. We obtain the quotient on adding the relation \(x = y\), to
determine the index of each one of the zones of the flattening,\textsuperscript{13} and we define as empties the zones of
even degrees, or this could be \(x^2 = 1\), in order that the peripheral zone would be empty, for greater
simplicity, as we indicate in fascicle #1.\textsuperscript{14} We obtain a spanning surface that corresponds to the
coloring of the odd zones, annotated x according to our previous quotient (\(x^2 = 1\)).

\textsuperscript{10} Essaim, p. 60
\textsuperscript{11} Essaim, pp. 105 and 118
\textsuperscript{12} Essaim, pp. 117
\textsuperscript{13} Essaim, pp. 122-126
\textsuperscript{14} Essaim, p. 90
(2) Search for the number of faces. Take up again the results of the calculus in the anterior stage (0), it is worth saying, before effectuating the quotient, to determine the fabric's number of faces.

The two central empty zones of the knot, annotated $x^2$ in figure 12, are indicated as $xy$ and $yx$ in figure 11. These zones are now holes, and the expression of each one of them is equal to 1. This way, from $\{67\}$ a first quotient of the fundamental group determinant of a knot's spanning surface (wholes/empties opposition), we obtain the indications necessary for establishing two new relations: $xy = 1$, and $yx = 1$, which corresponds to a new quotient realizable in the case of the results of figure 11.

We can then replace $xy$ and $yx$ by 1 in the expressions of the calculus of the knot's fundamental group. The relation of the group $xy^{-1}x^{-1}yx^{-1}y^{-1} = 1$ is reduced: $x(xy)^{-1}y(yx)^{-1} = xy = 1$, from which we deduce $y = x^{-1}$.

Each one of the whole zones of the spanning surface is designated, it already would be through $x$, already by the inverse of $x$: by means of this writing we distinguish the two faces of the surface that, therefore, is bilateral.

By making correspond a color to $x$ and another color to $x^{-1}$, we obtain a new coloring of the spanning surface that puts into relief the presence of two distinct faces. Really, $x$ is different from $x^{-1}$ ($x^{-2} = 1$). The reader can verify in the preceding calculus that the contrary equality, $x^2 = 1$, is not introduced in this stage (2), from the moment in which we return to the result given in figure 11.

We define with the same techniques the spanning surface of the second presentation of the trefoil knot and resolve its number of faces.

(0) Basic calculus. The calculus of the fundamental group gives us, from two generators, the demarcation next to the zones.$^{15}$

$^{15}$ Essaim, p. 105
(1) Distinction of wholes and empties. Take the coefficient to join the relation \( x = y \) in order to determine the index of each one of the zones, and define as empty the zones of even degree, or being \( x^2 = 1 \), in order that they would become empty like the exterior zone. We obtain a surface that corresponds to the coloring of the zones of odd index, annotated \( x \) according to the quotient \( x^2 = 1 \).

We have defined a trefoil knot's spanning surface, tied like a fustian. In order to determine its number of faces, we take up the calculus again from the start.

(2) Search for the number of faces. The central empty zone of the knot, annotated \( x^2 \) in figure 15, is annotated as \( xy \) in figure 14. Now, this zone is empty, for which we can think \( xy = 1 \) and replace \( xy \) by 1 in all of the expressions of the calculus of the fundamental group; what is important is a new coefficient of this group effectuated in the case of the results given by figure 14.

The relation \( xyx^{-1} = y^{-1}xy \) is reduced to \( x^1 = y^1 \), or it could be \( x = y \). What signifies that all of the whole zones of the spanning surface can be designated by \( x \): the fabric does not have more than a single face, it is unilateral. In effect, in this case, the relation \( y = x \), the simplified expression of the group relation, together with the relation \( xy = 1 \) from the quotient adopted here, gives \( x^2 = 1 \), which signifies that the group operation is involutionary and the fabric unilateral.
It is necessary to distinguish well the different calculations. In the first place, that of the knot's fundamental group (0); after, the quotient that we effectuate to join some relations that simplify the expressions of the fundamental group and those from which we obtain a knot's spanning surface (whole/empty opposition, \( x/1 \)) (1); and finally, \( \{69\} \) the transport from the resulting relations of the first quotient (that is to say, those which are written from the expressions of the empty zones, as such equal to 1) to the expression of the fundamental group's relations (here one relation only). By means of this second quotient (2) we get to change an indication relative to the number of faces of the spanning surface: it could be \( y = x^{-1} \), which corresponds to bilateral fabrics, or it could be \( y = x \), which corresponds to unilateral fabrics.

\( \text{a}_2 - \text{Edge number} \)

The edge of the spanning surface is composed by a set of separated circles. By circle we designate all closed curves that are a submersion of the trigonometric circle from analytic geometry (the classic “donkey bridge,”” \( x^2 + y^2 = 1 \)). To determining the edge number one has to count the number of circle components of the edge. This number of components of the edge, or edge number, is an intrinsic characteristic of the spanning surface. This will assume all of its importance in the next chapter, when we would be occupied with the Euler-Poincaré indicator and the genus of fabrics.

4. Reduction via drawing of a spanning surface to its intrinsic characteristics

Three operations enable one to reduce any spanning surface to the most simple form intrinsically equivalent to it (the same number of faces and the same edge number). One operation is extrinsic (it does not change the edge knot), two are intrinsic (they unmake the edge knot). By means of these topological transformations, continuous from one to the other, the main point is to conserve the (intrinsic) fabric without the knot, the knotting of the fabric being an extrinsic characteristic.\(^{16}\)

**Operation I: Deformation of edged surfaces**

The first operation consists in a change of the presentation of the spanning surface in the same way so that the other operations can be effectuated with facility. The deformation of the edged surfaces \( \{70\} \) is an extrinsic operation that makes appear the spanning surface considered as a set of ribbons (or

---

* The translator the Spanish edition notes here: “(Demonstration of the square's theorem of the hypoteneuse. Figuratively, an obstacle with what would only herd the ignorant.)”

\(^{16}\) If we denominate \( f \) for an intrinsic transformation, and \( F \) for an extrinsic transformation of the space that contains the strip of two semi-twists across the space that contains the strip without semi-twista, one must say that \( f \) is not a restriction of \( F \) (see the Introduction, pp. 38-9).
straps) of printed calico with folds or semi-twists or without them, which can be crossed over themselves and return to link themselves one to the other in a total connection. It is possible in that case to slide a ribbon along another, that is to say, to displace the intersection that they form.

Two cases can be presented: the displaced ribbon comes across neither with semi-twists nor with folds on the ribbon through which it is spread out...

It is significant that a same intersection of ribbons can be read in different pertinent manners among themselves.

...or, in the contrary case, a fold is going to be craeted upon the ribbon displaced by the passage of the semi-twist (or from the fold) of the ribbons that is stretched out.

{71}
Passage of a fold.

Resulting from the passage of a semi-twist.

The initial situation of this series of drawings can make room for another change of presentation.

**Operation II**: Suppression of the semi-twists of an even number

The second operation is intrinsic to the fabric: on a same fabric, consecutive semi-twists of an even number can be eliminated. This homotopy of the edge unmakes the (extrinsic) knot but conserves the intrinsic characteristics.

This way, a ribbon with an equal number of semi-twists (a whole number of twists) is intrinsically equivalent to a ribbon without semi-twists, like the relief effects of coarse printed cotton:
A ribbon with an odd number of semi-twists is equivalent to a ribbon with only one semi-twist.

**Operation III:** Above-below exchange of the fabric ribbons

The third operation is, like the preceding one, intrinsic. On the level of the crossing of two ribbons, these can cross themselves one to the other, as Nainsook makes possible, in such a way that what was below passes above.

One must specify that this homotopy of ribbons (or of strips) makes sense only pertaining to the same fabric ([they] are connected by their extremities).

Having defined these three operations, we continue on with two examples of the reduction of a spanning surface to its intrinsic characteristics.

In the first place we take up again the trefoil knot in its bilateral presentation whose characteristics we know, and after that the Borromean knot whose spanning surface we know and whose characteristics it is possible to know through the calculus that we can make, as we made it before for the two presentations of the trefoil knot.
Example of the trefoil knot in its bilateral presentation

The spanning surface is transformed into a presentation in ribbons that includes semi-twists.

Displace the link of one of these ribbons...

to pass a semi-twist, making itself a semi-twist.

Regroup the semi-twists that are paired.

In agreement with operation II, we eliminate the equal semi-twists.

We obtain this way an crossing of strips: a pierced torus.

The trefoil knot's intrinsic spanning surface in its bilateral presentation is a pierced torus.
Example of the Borromean knot

The spanning surface, without folds, of the Borromean knot.

Presenting it in a composition of ribbon crossings.

A first sliding along another semi-twist according to operation I.

Afterwords, a second sliding. In the two cases the passage of a semi-twist is made.

Fig. 25

Intermediate:

In which we recall that the pass from a semi-twist makes itself a new semi-twist.

Fig. 26
Regroup the semi-twists that were made on the same ribbons.

Eliminate the semi-twists by pairs, according to operation II (what remains is displaced).

To displace a ribbon along another, put into relief a ribbon loop.

*Intermediate:*

in which we show that a ribbon loop equals two semi-twists. This is seen to follow the mutual
Etlin's translation of *Fabric* - 8/11/9 – circulation is prohibited without my permission - (pp. 51-76)

dispositions of the two edge elements.

Two semi-twists, in that case, in lieu of the loop, that can be eliminated, according to operation II.

Displace one more fabric ribbon without finding semi-twists. Afterwords pass a semi-twist. A new semi-twist is produced,
Reduce the run of the fabric ribbon that carries two semi-twists.

Eliminate these two inverse semi-twists among themselves.

Leave a single semi-twist. Displace from the other side of the figure a fabric ribbon without semi-twist.

What gives this regular disposition, equals

A diagram made from a Mœbius strip with two holes.

The main point is the intrinsic structure of the Borromean knot's spanning surface.
The intrinsic characteristics of a knot's spanning surface are, therefore, the traces, the memory of the fact that there was a knot. On the fabric the knot is forgotten, like erasure by sponge; they leave some traces insufficient for reconstructing it; in effect, various different knots can have an intrinsic spanning surface that presents the same invariants. It remains in the case of this fabric a calculus in the knot genus.

We will show as we continue on in which manner the cuts on the spanning surfaces can be converted into a new invariant of the knot. These cuts demarcate holes.

In this chapter we have seen how the calculus in the fields of ex-sistence of these knots are replaced by coloring of lines, of a clear grey and of another darker grey, which substitute them with precision. These colorings translate the project of the driver grammer into the logic of the phantasm. In what remains of this work we will avoid returning to the algebraic calculus, and we will only employ the corresponding colorings.