The Ordinary Topology of Jacques Lacan*

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In 1679, Leibniz defined a new branch of mathematics under the rubric "analysis situs." The literal translation of this term, "study of place," situates this new discipline at the origin of topology.

However, topology is not truly given body until Euler develops his first theorem, in 1750. This theorem establishes a constant relation between the summits, surfaces, and edges of a convex solid; for example, the Platonic solids and the volumes of our everyday experience, like the pyramid, the cube, and the parallelepiped . . .: he thereby proposes some new solutions to some very old geometrical problems.

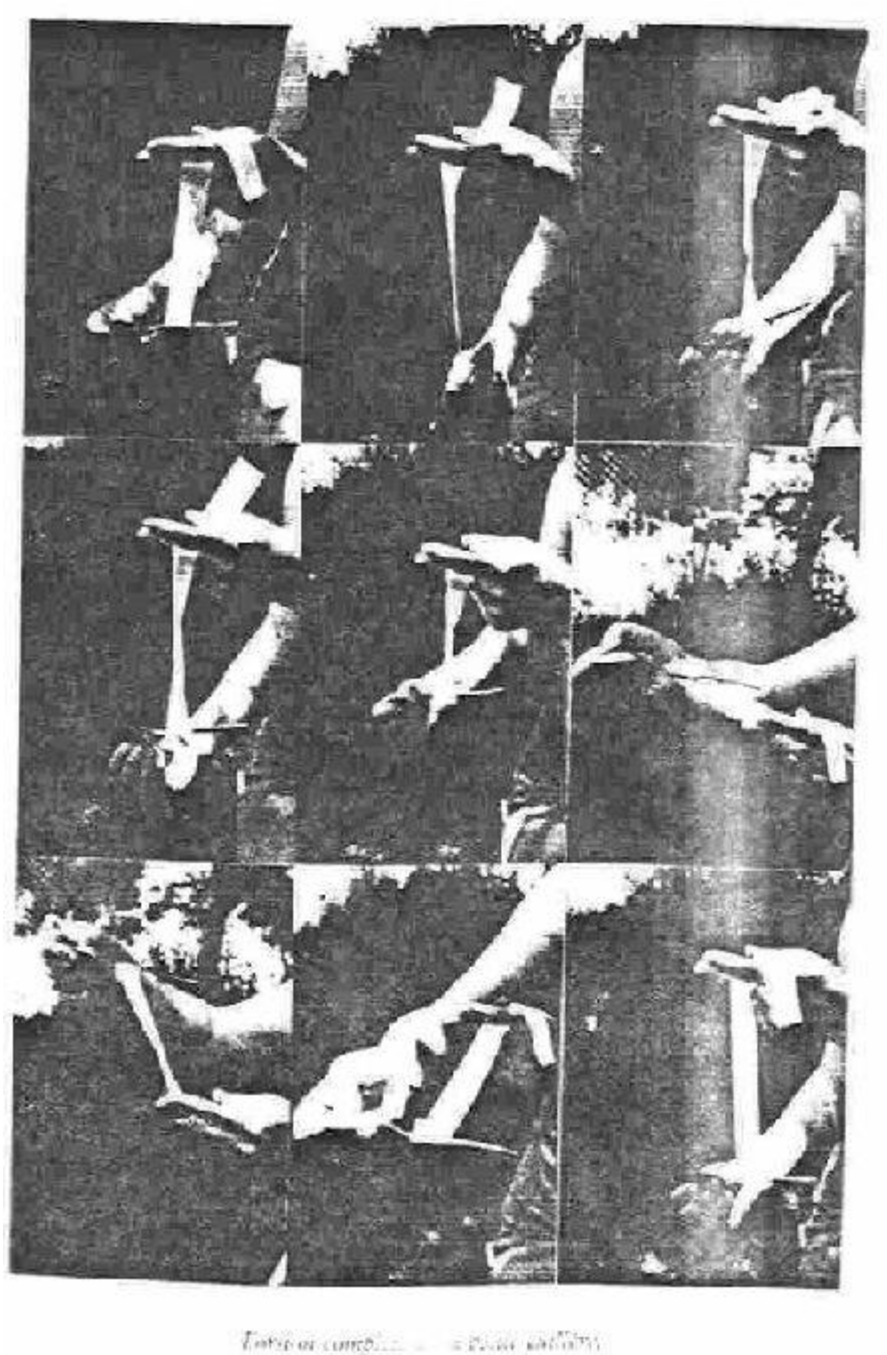
This theorem, the first of its kind, provoked arguments, and many mathematicians sought to define its limits.

In the perspective of these labors, in 1861, Moebius discovered the figure that would pass his name on to posterity: the Moebius strip. Unilateral surfaces were created, and would in one way or another devour, subsume under their laws, whole branches of mathematics. In 1874, Felix Klein and Shläfly put forth the idea that the space of projective geometry is Moebian. From here on, no one will speak of functions without reference to topology . . .

Thus, when Bourbaki, in 1948, newly formulates the ensemble of mathematical discoveries, they (*il*) will enumerate three, or more precisely two, to which a third group will be added: the structure of order, the structure of group, and the topological structures in reference to which they add in a note "that they escape the limits of the blueprint (*épure*)."[1]

The work that we propose here is situated at this "limit of the mathematical blueprint." It is a matter of studying the principal topological structures, beginning with the advances of Jacques Lacan in this domain

[1] Bourbaki, note 3 in "*Architecture des mathématics,*" an article in *Les Grands Courants de la mathématique*, presented by F. Le Lionnais (new edition, *Albert Blanchard*, Paris 1962).



Full-twist of the little spoon.

Chapter 1: Space, Structure

"In the beginning was space."

This paraphrase of both Saint John and Goethe puts the notion of space in relief; we will put it in the introduction of our study of Lacanian topology.

A little experiment will help us to grasp this notion. Take a small spoon and suspend it from a ribbon fixed at its top. This ribbon will materialize the tie of the spoon, our experimental object, in space.

If we give the spoon one complete turn around a vertical axis, it will reassume its initial position, while the ribbon, initially flat, has now taken on a helicoidal twist that reveals the effected operation.

After two turns, then three, the ribbon shows a double twist, then a triple one . . .

Thus, by glancing at the initially flat ribbon, we can know the exact number of complete turns effected by the spoon.

If we now give the spoon a turn, in the direction of a the hands on a watch for example, and if, maintaining it rigorously parallel to itself, we make it pass over the vertical part of the ribbon and it is returned to its first position at the bottom of the ribbon, we notice that the ribbon no longer shows any trace of the turn. Although the spoon has at no time changed its orientation, its movement alone has annulled the initial revolution (Cf. the photos at the beginning of the chapter).

This experiment shows us several things: to start with, it allows us an effective approach to the notion of space. The relation between the spoon and the ribbon is that of an object to its space. If you take a book in which you have printed the same movements, you will not see space appear; space is misrecognized in usual manipulations of objects. The spoon is an invariant object plunged into space. The relations between the movements of revolution (the turns) and of translation structure and define this space.

General topology is the study, the science, of these spaces and their properties.

It is not a question, as in classical Euclidean geometry, or even in that of Lobetchevsky or Riemmann, of producing a system of calculations and notations allowing us to situate an object and its movements in space. It is a question of describing space itself, while taking the invariance of the object into account. We must indeed remain conscious of this change in perspective in approaching topology, particularly that utilized by Lacan. Thus, Lacan could say, in *RSI*: "all space is flat; there are mathematicians who have spelled this out for us (*l'avoir écrit en toutes lettres*)."[2] How are we to understand this remark?

Space in itself does not encompass the dimension of depth, the famous third dimension. It is only for an object plunged into space, depending on how its movements unfold in time, that there is a before and an after, and consequently a front and a behind. Topologists have classically taken recourse, for manipulating this perception and its illusions, to the metaphor of the ant.[3]

Imagine an ant offered the same system of perception as a man reduced to its size. This animal walks along the surface of a Moebius strip, a flat surface of two dimensions, which is defined in this way in relation to its immediate environment. On the other hand, the horizonal point where the strip curves back on itself, and hints at its twist, always in relation to its immediate environment, is perceived as a depth. This depth is measured by the time that it will take for the ant to get to the point of the twist, and it will never attain it, since, as soon as it arrives there, another horizon will present itself, always as a third dimension, as a depth.

The "flat" is defined by the surface of a picture enclosed by an edge, and space is defined by the perception of depth. It is a question of a horizon that we know is not a limit, but,

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[2] Jacques Lacan, *Séminaire du 14 janvier 1975*, published in *Ornicar* n. 3.

[3] An ant that we find diabolically represented in some pictures on the cover of issues of *Quarto* (the revue of *la cause freudienne* in Belgium).

topologically, of the time that it takes to get there.[4] Clinically, this reality of our space is essential for conceiving of the state of the world experienced by the psychotic, of which one says that it is without limit, in other words, without a temporal dimension. Time, from the point of view of topology, is the dimension of space considered as flat, as a surface. Thus, in our experiment with the little spoon, the relations between the movements of revolution and translation structure this space which is ours, and define it. There exists, moreover, a property of our ordinary space thus presented that leads to differences in space, differences in structures. To demonstrate this, we are going to take an obligatory detour. We are going to try to demonstrate what is, according to us, an error of Jean-Marc Lévy-Leblond, which will bring us back to the little spoon already evoked.[5] Let us cite, to start with, a fragment of his procedure: "the astonishing thing is the completely particular role played by the revolution of two turns." If we redo this experiment again, after an initial revolution of a single completed turn, we will quite clearly find the ribbon affected by a turn in the opposite direction from the first. Without a supplementary revolution, and by simple parallel movements of the spoon, it is possible to modify two by two the number of the complete turns exhibited by the twist in the ribbon.

"There is a difference of an essential nature between the rotations involving an even number and those involving an odd number. Thus, one can say that it is not sufficient to give an object a complete turn of either three or five . . . bringing it back exactly into the same conditions of relations with its spatial environment. To do this, we must effect two (or four) complete turns; the double turn is found to be the basal unity."

Jean-Marc Lévy-LeBlond continues: "There we have a fundamental property, at once extraordinarily simple and perfectly enigmatic, of the space where we live. At our level, this property, to my knowledge, has no other manifestations than of permitting certain tricks to professional magicians. On the other hand, it has a considerable importance in microscopic physics. Quantum theory founds the existence of two distinct classes of fundamental particles on this property: 'bosons,' described for mathematical beings by a single revolution left unchanged; and "fermions," which require, on the contrary, a double revolution to recover their initial description. If we now recognize that the 'fermions' obey the 'Pauli' exclusion principle, which, applied to electrons, explains what is essential in the properties of ordinary matter, we see that it is no longer a question of simple academic curiosity. In truth, it is precisely the experimental discovery, then the theoretical understanding of 'fermions' at the quite esoteric level of fundamental quantum physics, that has allowed us to go back to the source of their strange characteristics, found finally in the unexpected difference in nature between a single and a double turn."

What is at stake in this experiment is not meager and, as it is presented, it seems to involve a mistake. The error is situated at the level of the naming of the particularity of our ordinary space, and, in this argument, the Moebius strip is grossly misunderstood.

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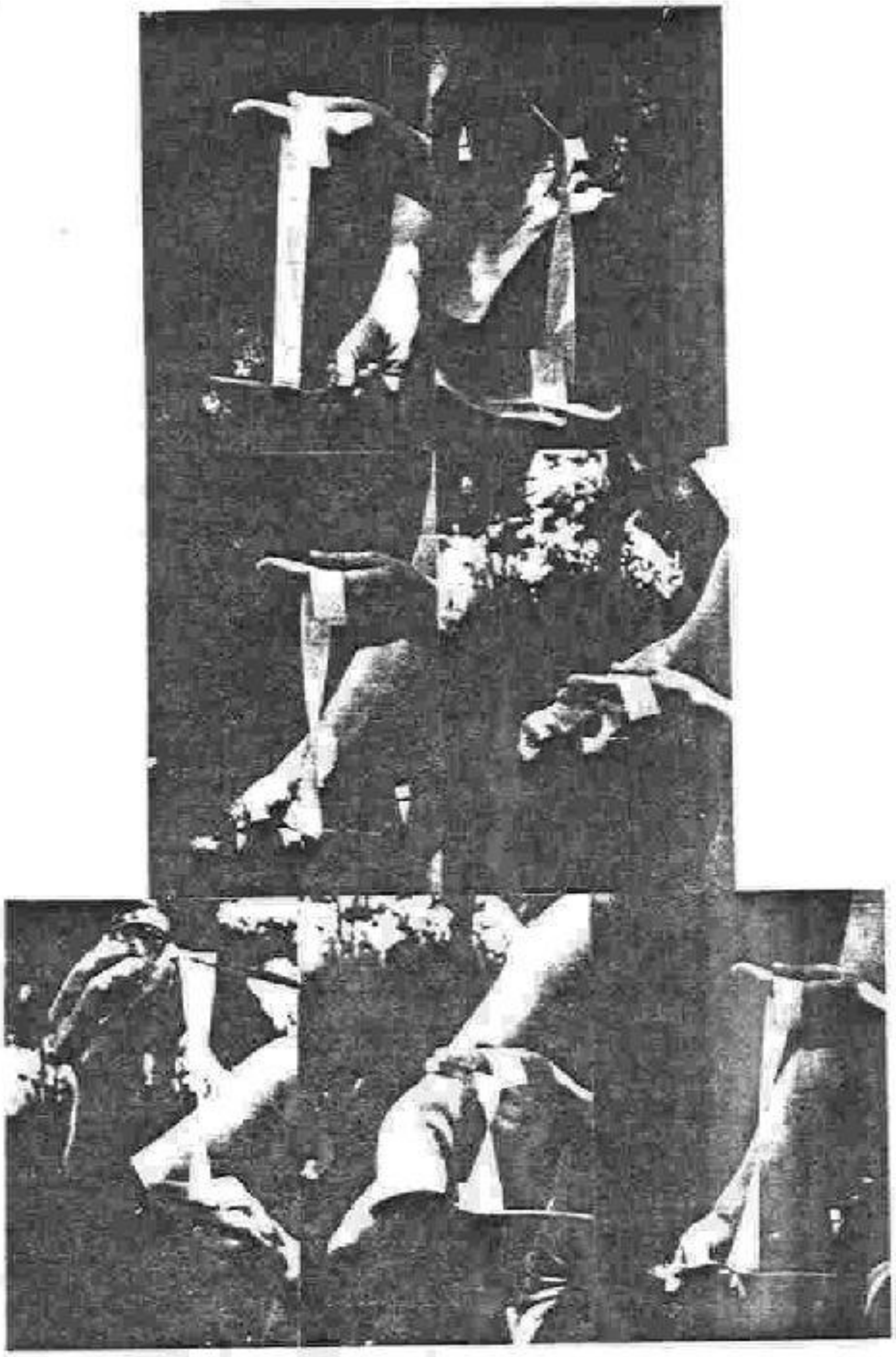
4 In *L' Empire des lumières*, in *Traverses*, September 1978.

5 Jean Claude Terrasson has published a quite illuminating text on this question in the revue *Littoral* n. 5, June 1981.



Demi-torsion de la petite cuillere.

Half-twist of the little spoon.

If we again take up our demonstration, we must notice the difference between the full-twist and the half-twist. J.M. Lévy-Leblond speaks of the revolution of two turns, when it is, in fact, a question of a half-twist. A turn of the little spoon gives a full-twist to the ribbon, that is, two half-twists. The translation in space, above the place where the spoon is connected to the ribbon, always modifies the ribbon by two in two half-twists.

The little spoon can make three complete turns: this gives three full-twists of the ribbon, that is, six visible half-twists, and three translations, to return to the first position of the ribbon.

On the other hand, if we give the spoon a half-turn, after the translation in space, the ribbon takes on a complete twist. Thus it presents, at the end of the operation, a half-twist in the opposite direction from that created by the first half-turn. For the turn of the little spoon, the basal unity is a half, as it is for the ribbon, but the unity is 1 for the translation. It is impossible, in ordinary space, to bring forth, by translation, a half-twist. It is here alone that we find the radical difference between even and odd. Three half-turns are fundamentally different, for the ribbon and for the materialization of space that it effects, from two or four half-turns.

This experiment makes appear with striking clarity the Moebian space, or, more precisely, the space of the projective plane. If the space in which the ribbon is plunged were Moebian, a translation would make a half-turn disappear. For translation also, the basal unity would be the half-turn. One can demonstrate this by looping a Moebius strip through the little spoon.

We see how these relations structure space, rendering it materializable, while it most often escapes our perception. The specific object of topologists is this notion of space and the relations that structure it.

It is in this measure that topology concerns psychoanalysis. Psychoanalysis is, in fact, a study of structure disencumbered of a singular, substantified psychic object. The general tendency of thought is to give body, subjectivity, to one or more of the concepts of our discipline, for example, the subject or the unconscious. The subject is not the object of psychoanalysis, just as the ant and the little spoon are not the objects of study of topologists. They are only interested in their appearances, their trajectories, and in the possibilities that allow them to describe a particular space. So it goes for the soul, a notion that serves as an exemplary representation of this tendency toward subjectivation: for the topologist, the study of its depths only puts in play questions of passages through space.

Moreover, topologists have defined a notion of space that is identical with the structure utilized by the human sciences. Topology proves to be the study of the structure in play in these sciences, as will be demonstrated after the fact by the discoveries of Jacques Lacan.

We can show this by drawing on the ethnographic studies of Lévi-Strauss, the importance of which for the introduction of structuralism into the human sciences is well known.

This is the case in the study Lévi-Strauss conducts on the matrimonial system of the "Kasieras" (Who has the right to marry whom?).

He defines the structure of this system starting with a division of the group of matrimonial classes into two patrilinear halves, a division that is itself "perpendicular," he says, "to the division into two matrilinear halves."

The word "perpendicular" does not at all refer to a precise mathematical definition, but rather to an intuitive vision of space, to a schema that would represent this structure on two perpendicular lines. Moreover, without entering into the details, Phllippe Courrèges has shown that the true mathematical concept that would be pertinent here would be that of "product."[6]

We can also cite the work of Henri Pradelles on the kinship system of the Trobriands (a classic object of Malinowski's studies). He shows that the concept of a "duality of cuts" is the

[6] Phillippe Courrèges, in *Anthropologie and Calcul*, U.G.E., Coll. 10/18, Paris, 1971.

best approach to this particular ethnological structure.[7] Without resorting to a forced analogy, these examples show how, from the origin of the work of the structuralists, the conceptual necessity of bringing in topology comes to light. At its limit, as soon as a "schema" takes on an explanatory or even a didactic value, topology intervenes as an epistemological foundation to the knowledge (*connaissances*) brought forth by this schema. It is wholly to Lacan's credit to have sought to pinpoint (*cerner*) this specificity of topology and to have indicated what its usage could be in the human sciences.

As for Lévi-Strauss, he did not seek exactitude in this domain, too conscious of the distance that separated his work from mathematical formulations. For instance, in the preface of *The Raw and the Cooked*, he writes: "It better that no one scruple over the very loose acceptations that we give to terms such as symmetry, inversion, equivalence, homology, isomorphy . . . We utilize them to designate large packets of relations of which we perceive confusedly that they have something in common." The allusion to a "confused" perception is appropriate: it appears to us, in fact, not only that topology is confusedly present in the work of Lévi-Strauss, but that it is also a putting to work of this confused perception of structures. Saying that he does topology without knowing it is in no way impertinent, if we consider, *apropos* of common themes like the Oedipus complex, all of the work of Lacan, which consisted in clearing up this confusion from a certain perspective.

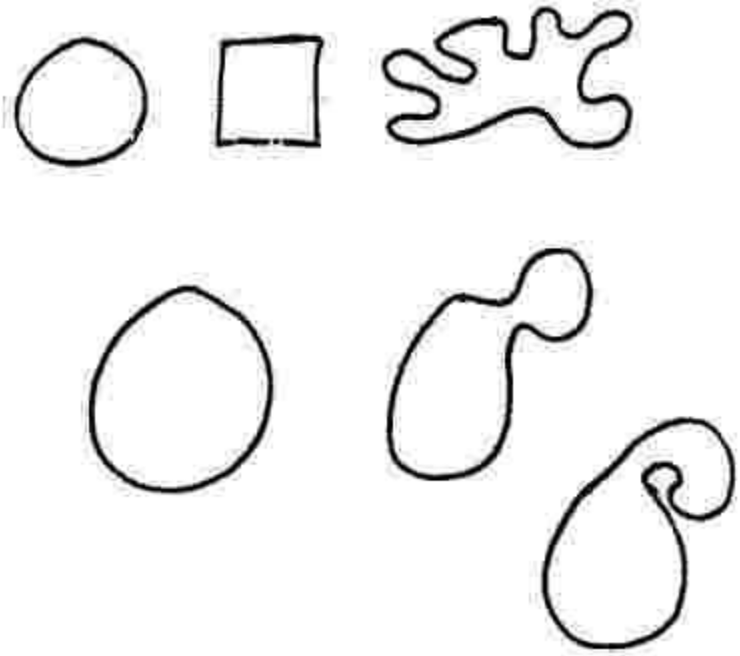
Also, the aim of our work is not to know how mathematicians came to be interested in this study of space and spaces, although renowned topologists like Poincaré speak with respect of the "geometrical intuition" that allows us to confusedly perceive these "large packets of relations" . . . On the contrary, it is a question of situating, beginning with what is at stake in psychoanalysis, the topology of spaces. For this, we will take support, not from the requirements of mathematical discourse, but from the necessities internal to analytic discourse. Topology clarifies the notions on which the psychoanalytic treatment reposes. The psychoanalyst has a means for establishing what is at stake in psychic suffering by recourse to topological structures.

We must now approach this topology directly.

Treating of the notion of space, Topology is interested neither in the metrical nor in proportions. Based on this fact, equality is defined as the possible trajectory from one presentation to another. Two figures are called identical if it is possible to pass from one to another by a continuous transformation.

From this perspective, these objects are the same (we already grasp the importance of the drawing).

[7] Cf. the article of Charles-Henri Pradelles of Latour, *Littoral*, nos. 11-12, February 1984, Paris, "*la Parenté trobiandaise reconsidérée*."

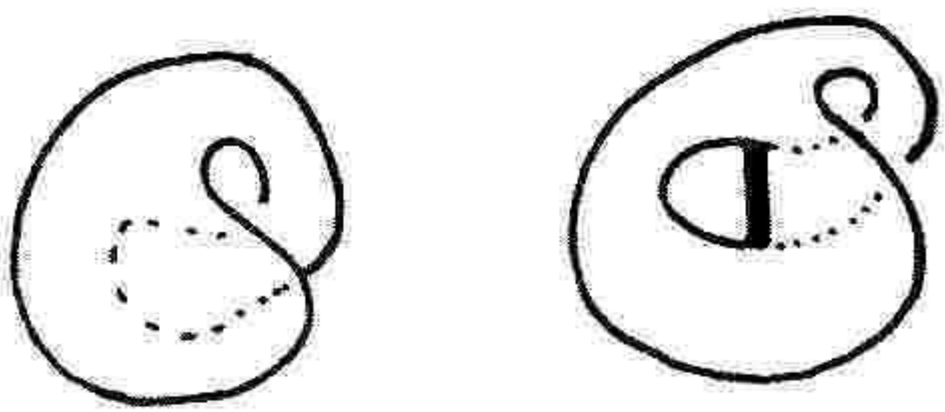


A surface like the disk can vary continually without modifying its structure.

However, at a certain moment there is rupture and a passage is effected from one structure to another.

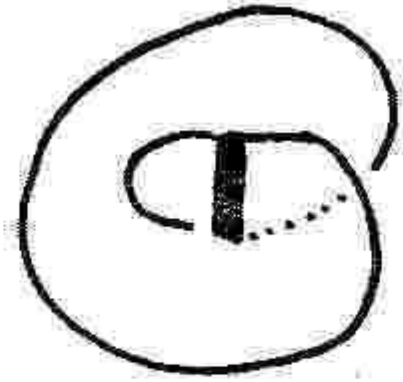
This transformation has only been possible by the slow, continuous preparation of the surface.

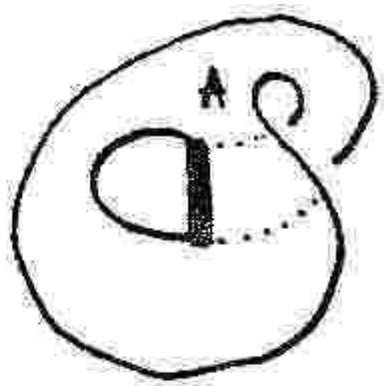
Thus, on this disk, one part of the surface can pass beneath itself, and it is possible to make reappear the portion that has been slipped beneath. We have thus created a line of



intersection. This line of intersection signals the passage of the structure of a submerged surface to an immerged structure. We have radically changed a domain, passing from submersions to immersions.

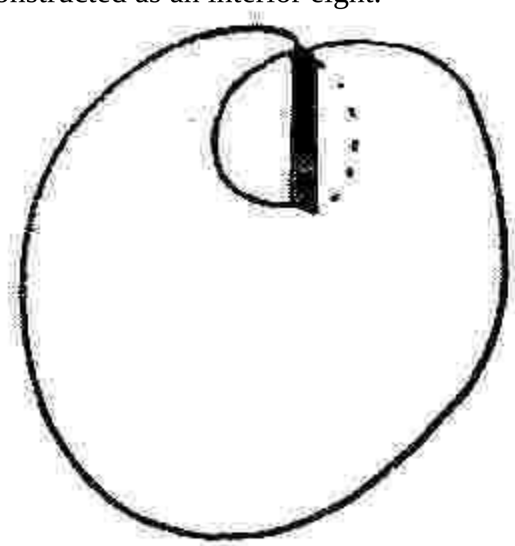
The hole that we designate at point A can then be reduced to a point: this is equivalent to the phenomenon of the twist. This surface can be seen as a disk that can be twisted back on itself. (The twist has a quite distinct status that will be the object of a whole chapter.)





By this phenomenon of intersection and of disappearance of the hole, there is an obvious transformation of the structure.

It is intuitively perceptible that we have changed its space (we are already in the space of the projective plane). We can then make the interior line evolve into this point A. We thus obtain the immersed disk, constructed as an interior eight.



This exercise clearly shows why topology has been called "rubber geometry."

This example allows us to render easily sensible the game of transformations in topology. There exists a whole dialectic between the preparation of a surface by continuous transformation of its drawing and the brusque appearance of an event, of an act, of an operation; of a changing of the structure of this surface or of its space of submersion. This dialectic, between the continuity of the identical to the identical and a structural rupture, is essential to our approach to topology. It allows us to understand what conditions the completely particular relationship to time and to the scansion that we know in the treatment. Often, an interpretation only has an effect after a long series of sessions that have done no more than make the presentation of the symptom evolve, without modifying its structure.

Once we have set forth these preliminaries, we can take on the study of topological spaces. We will then see how the Borromean knot will formalize the recourse to surfaces. Not until the end, in a supplement that will take the form of an index, will we give the mathematical definitions (signaled in the text by an asterix) that serve in the elaboration of this presentation of the topology proper to Jacques Lacan.