

Two linked torii

Chapter 3: The Torus

In the text of the "Rome Report" of 1953, "Function and Field of Speech and Language," we find under the pen of Jacques Lacan a reference to the topology of the torus.

To speak of the nature of the subject proper to the unconscious, Lacan situates himself thusly:

"When we wish to attain in the subject what was before the serial games of speech and what is primordial in the birth of symbols, we find it in death, from where its existence takes all that it has of sense. It is as desire for death, in fact, that it affirms itself for others; if it is identified with the other, it is in freezing him in the metamorphosis of his essential image, and no being is ever evoked by it except among the shadows of death."

"Saying that this mortal sense reveals in speech a *center external to language* is more than a metaphor and manifests a structure; this structure is different from the spacialization of the circumference or the sphere where one is pleased to schematize the limits of the living being (*du vivant*) and its mean (*milieu*): this structure corresponds instead to the relational group that symbolic logic designates topologically as a ring."

"In wishing to give to it an intuitive representation, it seems that, rather than to the superficiality of a zone, it is to the three dimensional form of the torus that we must take recourse, inasmuch as its peripheral exteriority and its central exteriority constitute a single region."

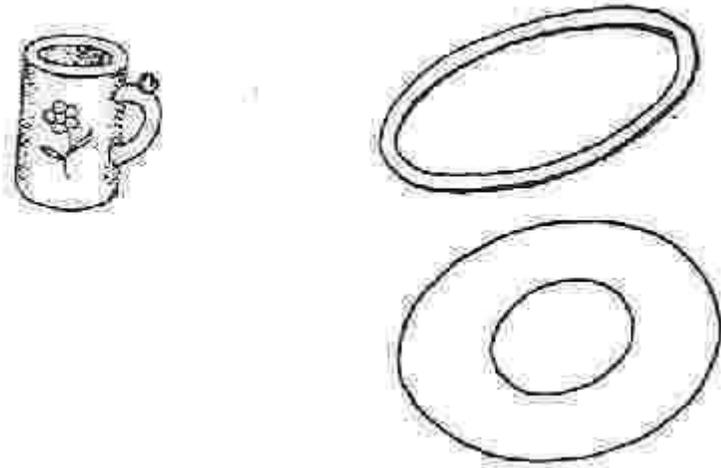
This citation is accompanied by a note from 1966 that reminds us of a usage of topology.

Our attention should be drawn to several terms that enclose some notions essential to the psychoanalytic cure. At issue in this "what is primordial to the birth of symbols" is what Freud calls the identification with the father of the primal horde, with the archaic father, with the dead father. At the dawn of the birth of the subject there is identification. This is the great question that the torus and its topology allows us to pose in clear terms.

From the "Rome Report" in 1953 to his *séminaire* of 1976, Lacan refines the formalization of this question. At first, he speaks of an "intuitive representation," and then the model gathers so much strength and conviction that once again the formula "topology is structure" imposes itself. Lacan does not have to articulate the relations between identification

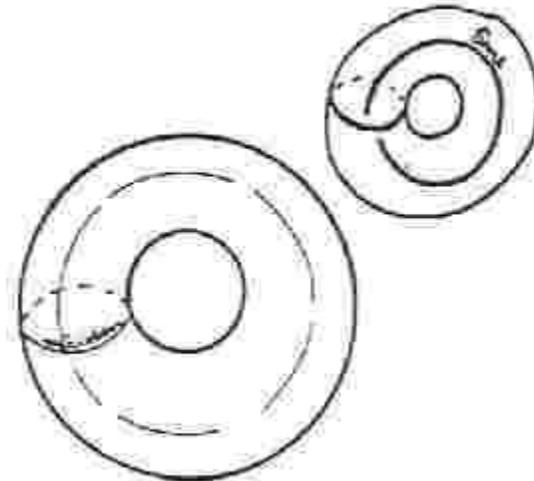
and the turning inside-out of the torus; the one is equivalent to the other. But we are getting ahead of ourselves.

The torus gives us an apt representation of this relational group for which the center and the exterior are one and the same space. The surface of the torus envelops an interior space and detaches it from the exterior, at the cost of a center that remains exterior. A torus is defined as a surface without edge, and is equivalent in this sense to the sphere, but its center is empty. The best physical approximation is the buoy or the inner-tube. A ring can also figure it if one takes into account the material sameness of a cup with its handle.



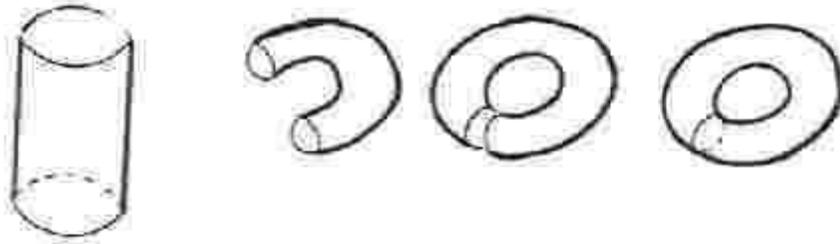
The lines drawn in this representation are the folds in the surface. We must imagine a flattened buoy.

We can obtain a torus by combining a circle with a circle (the Cartesian product of $S \times S$).

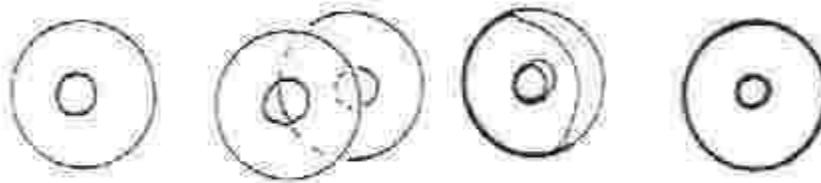


One circle is called the soul of the torus, the interior, or more precisely, the emptiness in the interior of the torus. The other is a little circle or meridian circle.

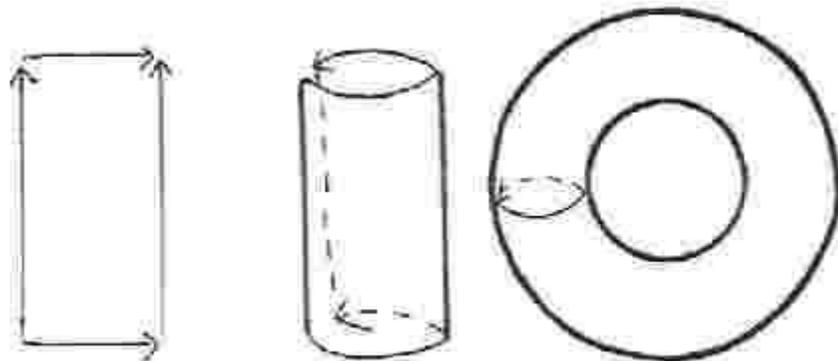
We can construct a torus from a cylinder: it suffices to start by transforming it into a handle by curving it lengthwise and joining its ends.



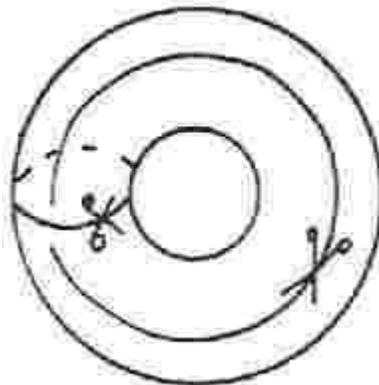
The torus can also be constructed from two crowns (*couronnes*): it suffices to join them along their edges.



We can represent this operation beginning with a rectangle, on which the arrows indicate the directions of the joins: we first obtain a cylinder, and then, by joining its ends, we obtain a torus.



These latter drawings allow us to define the torus as an edgeless surface that two cuts do not make disappear and do not divide (at issue are two dual cuts* that meet at a single point).



In this instance, Lacan defines the torus as an organization of the hole.¹ He expresses himself as follows: "These two holes isolated on the surface of a sphere are those which, lined up with each other (*rejoins l'un à l'autre*) and then very much extended and conjoined, have given us the torus." Around the interior hole "buried in the surface," and the axial hole, which Lacan calls the "current-of-air hole," a surface is organized. It is a holed interior that misses the center.

The torus, an edgeless surface, delimits an interior and an exterior with this particularity of having an "exterior" center. This center is holed; it allows for a knotting.

Lacan utilizes this surface structure in reflecting on the great question of identification. In 1976, he expresses himself in this way: "What relationship is there between our having to admit that we have an interior, which one calls what one can, psychism for example--we even see Freud write endo-psychism, and it does not go without saying that the psyche be endo, that this endo must be endorsed--what relation is there between this interior and what we currently call identification?"

Identification is an answer to a question: how does something from the exterior become interior, exterior, and still central?

Let us recall that for Freud there are three identifications:

--The primordial identification, said to be "with the dead father"; this identification results from the love dedicated to the father; it is responsible for the introduction of the symbolic.

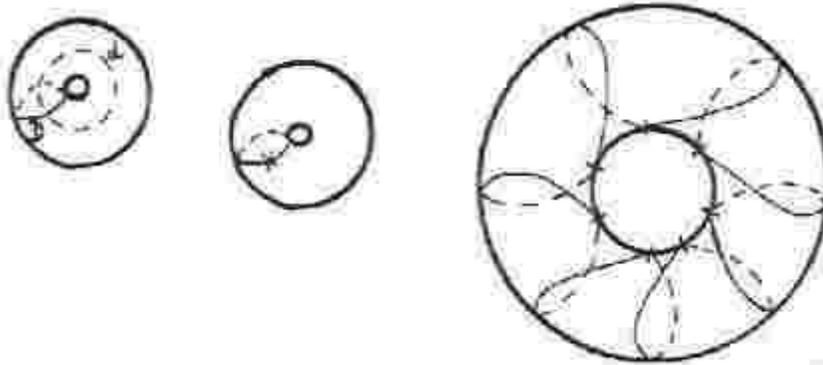
--The identification with the unary trait, where it would not at all be a question of love; Freud puts this identification at the foundation of the constitution of masses. Lacan illustrates it with the example of the Führer's mustache.

--The identification that implies a participation: it is pinned to the term hysteric, the identification with the desire of the Other.

How does the torus account for identification? This object offers a support that allows us to perceive the implications of this term that has become commonplace without, however, becoming more explicit. Lacan begins by drawing on a very particular phenomena of toric transformation, "the turning inside-out." But before taking on this notion, we must clarify certain terms that Lacan utilizes in reference to this surface, which allow us to establish the relation that unites desire and demand.

Let us designate with letters desire (d) and demand (D). On the surface of the torus exists a trajectory following a meridian circle. It closes itself in a buckle. On the other hand, if this trajectory around the torus misses its starting point, closing itself without intersecting itself, the loops are multiplied and the trajectory also completes a long longitudinal turn. It also turns the length of the torus's soul.

¹ *Séminaire on L'Identification*, May 23, 1962.

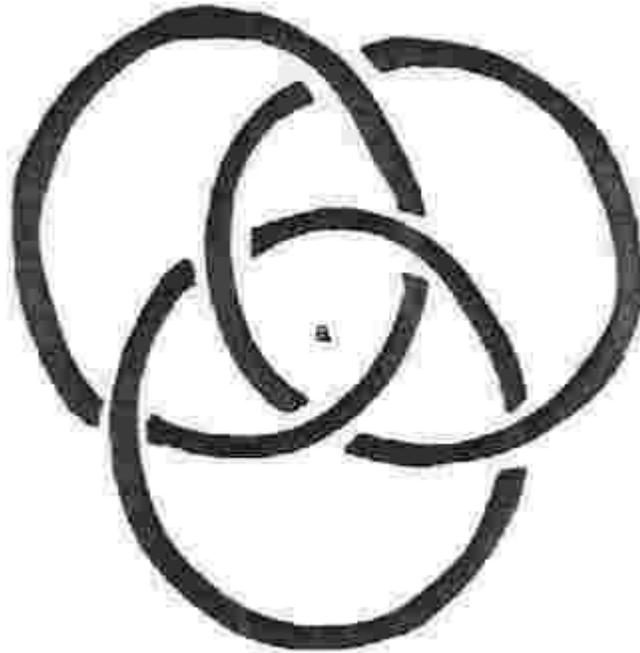


This trajectory encircles the torus's central hole in a manner that we could call pointillistic. In doing so, it describes a supplementary turn around the hole. This additional turn is forgotten; and anyhow, how are we to count it?

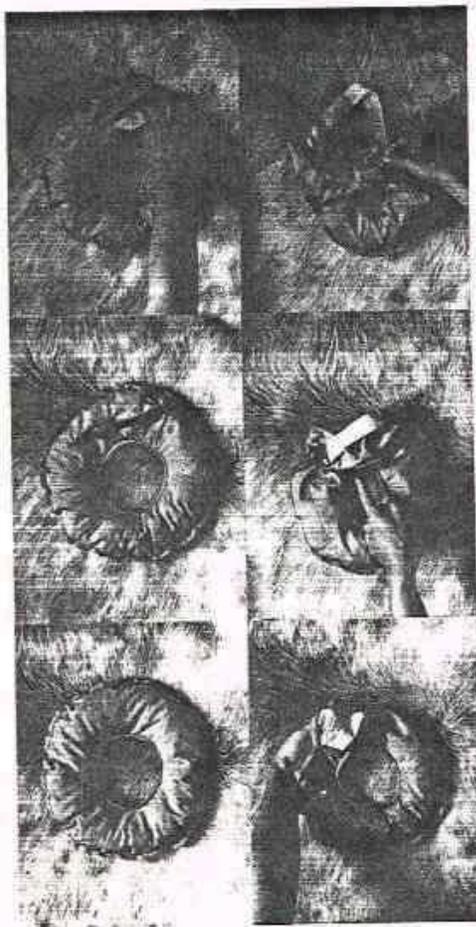
The turns succeed one another and are counted; they are identical, without the possibility of counting the additional turn completed around the central hole. Here is illustrated demand and its fundamental repetition, a repetition effected in the misrecognition (*méconnaissance*) of its expression of a misrecognized albeit essential desire. We have thus defined the "one-too-many" (*l'un-en-trop*): this forgotten circle of longitude, which is properly speaking what Lacan names desire.

This progress allows a fundamental aspect of the misrecognition of desire to appear, one summed up by the importance of the central hole: demand repeats itself and designates the object as lacking. Described in this way, this object is always missed, with a missing that is nonetheless structural, tied to the progress of demand and necessary to its repetition. This central hole is also in communication with the exterior, and Lacan utilizes these properties to define two distinct positions of desire in relation to demand.

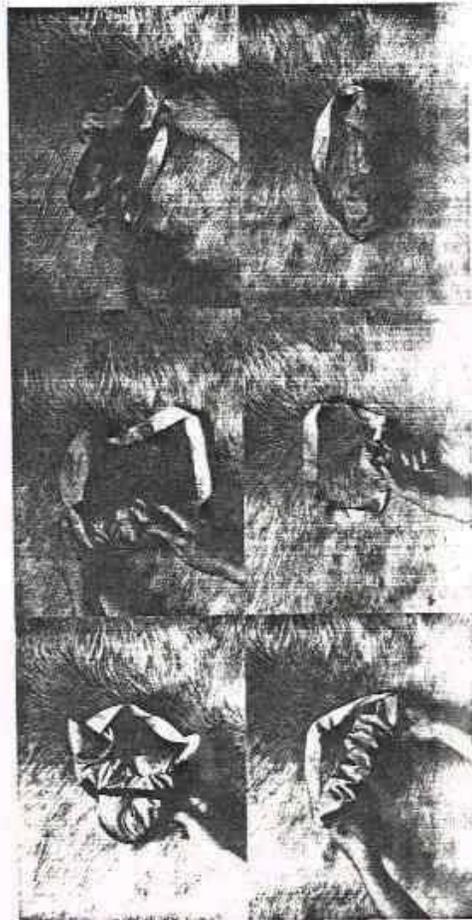
On the one hand, it is "beyond" demand--"it transcends it, goes farther, and is in this regard eternal." Demand, in articulating desire to the conditions of language, expresses itself through signifiers that betray its true aim. In a way, the missing is fundamental to demand; the figure of the object (a) is profiled in the central void. Lacan later describes this object as taken between the three rounds of the Borromean knot, RSI. Remember that these three rounds are torii; they have the consistency of the cord, "these are gut-torii (*tores-boyaux*)."



On the other hand, desire is "within"; the central void communicates with the exterior. Demand recalls the radical "lack in being" that subtends desire. "Desire hollows itself within, in that, as an unconditional demand for absence or presence, it evokes the radical lack in being in the three figures of the nothing, which founds the demand for love, of hatred, which tends to deny the being of the other, and of the unsayable, which ignores its request." We see appear in this sentence the three passions that Lacan situates at the level of being and not of the object. It is a question of hatred, love, and ignorance.



Retournement du tore.



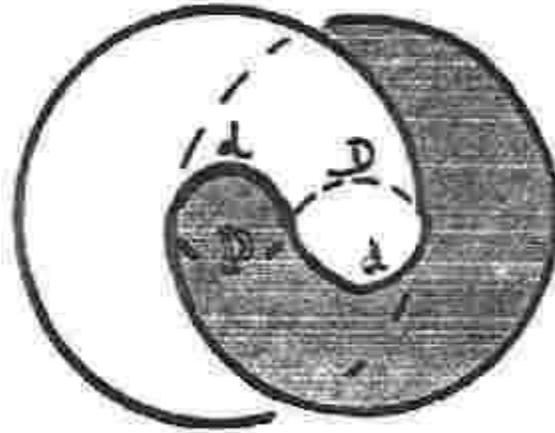
Retournement du tore (suite).

The turning inside-out of a torus

The demand that circles, borders the "lack in being," the nothing of the universe, creates, by its repetition itself, a surface separating an interior and an exterior. This structure accounts for the birth of a subject of the unconscious. The within of demand also introduces us to what there is in it of a knotting to the other. Love, hatred, and ignorance concern the other in his being. Remember that the subject who enters analysis puts himself in the position of "he who is ignorant" of what he says.

Lacan supports the neurotic dialectic between the subject and the Other with the knotting of two torii. In this knotting, the desire of the one is isomorphic with the demand of the other, and the central void serves only for the knotting of the two torii.

(Cf. the photographs.)



At issue is an essential articulation for entering the problematic of identification. An object demanded by the other, the mother, the primordial Other, finds itself in the position of object of desire for the subject. This articulation allows for a new envisioning of mother-child relations, which are relations of dependence, certainly, but do not belong to a symbiotic confusion or infra-verbal communication.

The signifiers that become unconscious are tied to signifiers witnessing to the moment of access to language. Here is fixed the structure of the fantasy, within demand, which is the mode of appearance of the Other. The fundamental fantasy situates (*cerne*) the moment of separation from the real experience, linked to the present demand of the other, and its hallucinatory revivification. It constitutes the separation between the object that fills and the sign that inscribes at the same time the object and its absence. At issue is the putting in place of the conditions of speech, the structure of which gives meaning to the aphorism: "the unconscious is the desire of the other."

Thanks to the notion of the turning inside-out of the torus, Lacan once more makes his thoughts clear. Turning the torus inside-out consists in making pass to the exterior the face which was on the interior. This operation can be effected thanks to a cut, to a hole. We then observe an astonishing phenomenon: the circles of demand and desire exchange positions. The meridian circle becomes a circle through the soul of the torus. However, the central hole remains the same.

At the physical level, the experience is simple, but its writing or drawing is very difficult, because the conventional lines of the curves disappear. This operation brings to light the purely conventional aspect of drawing.

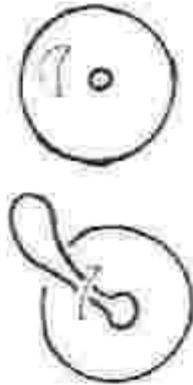
At the end of the process, the torus remains the same, but its writing is different.

The photos depict the whole of the operation, but we are going to study in detail these difficulties of the writing of it.

The turning inside-out illuminates how the drawing no longer maintains the illusion of a representation of the real that escapes writing.

We must begin with the classic drawing of the torus in which the lines represent the curvings of the surface: on this drawing we make a cut.

For the sake of greater simplicity, a hole will suffice.



A hole in the surface, a rupture, is of another nature than the central or interior holes of a torus.

Then we begin to pull the surface through the hole.

We turn the torus inside-out like a glove, or, better yet, like a poncho and its lining.

We see that the space of the central hole is going to become the internal space.

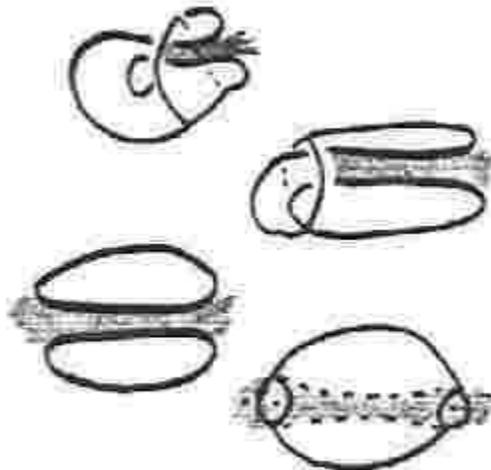
It is a question of a turning inside-out that remains in the domain of plungings (*plongements*). In general, mathematicians turn the torus inside-out at the cost of an intersection of the surface and enter into the domain of immersions.* The trajectory then passes through the Klein bottle, which is a unilateral continuous surface, and therefore puts interior and exterior in communication. The cut is here much more economical for operating the same suppression of the edge.

Moreover, in the field of psychoanalysis, the cut has different operator dimension than the intersection, since it refers to interpretation in the cure and, more generally, to the act of speech.

We can also make the following monstration:

The hole is open to show again what becomes of space. We can materialize this outcome by knotting a cord to the initial torus, which is what the photographs show.

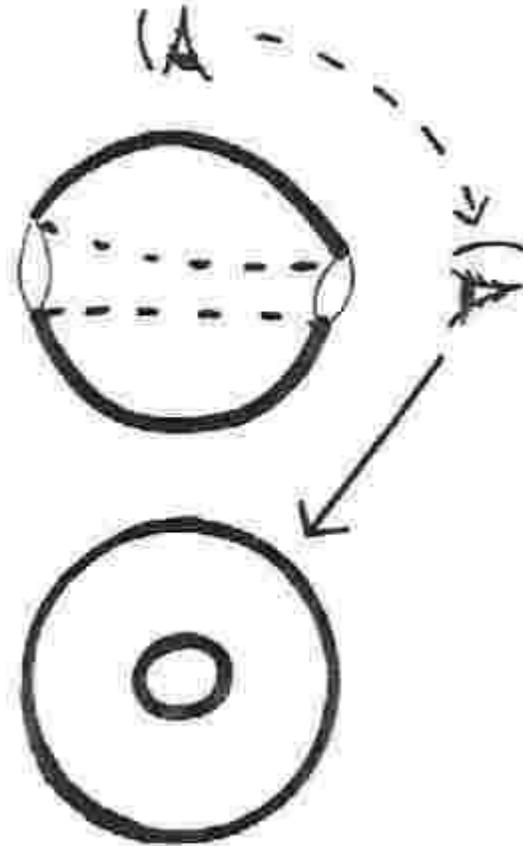
We then obtain another writing of the torus:



This drawing is represented as a sphere with a tunnel, with two openings, but then the lines that represent it are not creased. Lacan calls this presentation the "torus-cudgel" (*tore-trique*). By a rotation of a quarter of a turn on the part of the observer, we fall back into the classic drawing of

the torus. At issue is the same torus that we began with, but now its internal face is on the exterior.

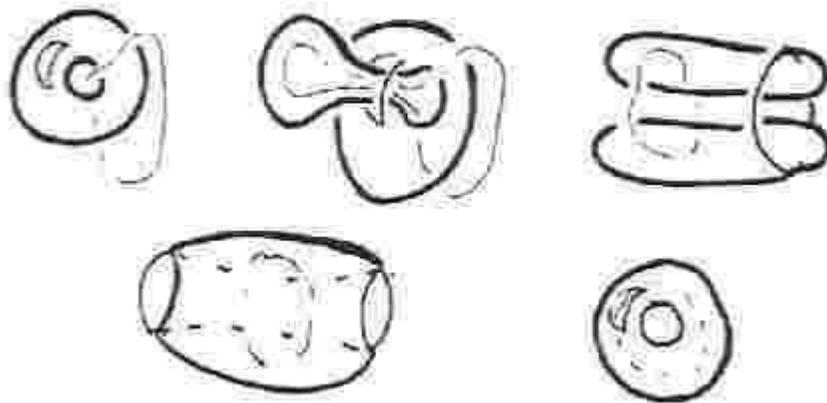
In these representations, we find again the importance of the twist. The subject of perception makes a quarter turn, as is symbolized by an eye in these drawings:



We see how the twist is an extrinsic characteristic of the surface, which only appears to an external gaze. (This question will be addressed in a whole chapter of this study).

Let us only note that it is a question of a quarter turn, of a half of a half twist. We have seen how the half-twist is the unit of counting in our space (Cf. the little spoon).

A thread that we have now represented in the following drawings materializes the transformation of the meridian circle into the circle of the soul. A circle of demand becomes a circle of desire.



This process accounts for identification, for the transformation of an object of love into a trait of the Ego, a trait with which the Ego identifies, or, rather, identifies its desire.

The drawings also allow us an approach to the scenario of the turning inside-out of two linked torii. It suffices to give to the thread the consistency of a cord or of a tube; we see how one can turn a torus inside-out and see the torus linked on its interior.

Or, visa-versa, the interior torus, from the moment when one turns inside-out the torus that encloses it, will be knotted to the first, as the place of the neurotic dialectic with the Other. (Cf. above).

We see now why a mechanism like this is important for accounting for the process of the development of mother-child relations, and how identification is an outcome of this attachment. We remember, for instance, how Melanie Klein established the mourning necessary for the separation from the primary object and the structuring role that it brings into play in the "depressive position."

It is possible for us now to return to the three Freudian identifications, to envisage them in terms of three scenarios of linked torii turned inside-out;

--a single cut, a single turning inside-out.

--a cut in the torus that we have just come to in our drawings and a turning inside-out;

--a cut in each torus, and two turnings inside-out.



With these schematizations, Lacan tries to support diverse Freudian identifications. He asks the following question:² "How are we to designate in a homologous fashion the three identifications distinguished by Freud: hysterical identification, the loving identification said to be with the father, and the identification that I will name neuter, which is neither the one nor the other, the identification with a particular trait, with a trait that I call no matter which, with a trait that is only the same? How are we to distribute these three inversions of torii, homogenous in their application (*pratique*) and which, moreover, maintain the symmetry of one torus with another?"

In the following seminar, Lacan takes up this problem in different terms: he endeavors to knot these identifications with the function of the unconscious.

It is in fact appropriate to situate the unconscious and its effects of speech in this problematic. "The torus can be cut into a double Moebius strip, and it is this that gives the image of the link between the consciousness and unconscious."

Elsewhere Lacan declares: "consciousness and the unconscious are supported and communicate by a toric world."

Toric space has many relations with the Moebius strip, but this not immediately felt.

By wrapping a Moebius strip with four half-twists all the length of its ring as a doubled covering (*revêtement*), we in fact find again a unilateral Moebius strip with a half-twist,* which, moreover, founds what there is of a hole. The hole is Moebian inasmuch as, if a surface has a topside and an underside, it has two holes, that in the topside and that in the underside. The Moebius strip, since it joins the underside and the topside, is a hole.

² Seminar of November 16, 1976, *Ornicar* nos. 12-13, *Le Seuil*.

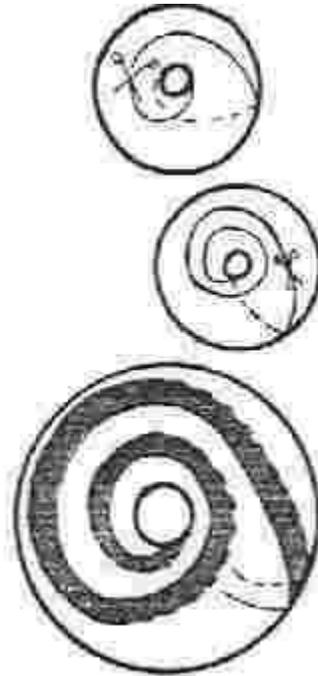
Some such layout is primordial for the functioning of the unconscious, for it allows us to situate its principal characteristic. It is a question of a hole, but of a different nature than the axial hole in the torus, although it entertains with it some particular relations.

How can a unilateral Moebius strip be cut from a torus?

The torus is a continuous, bilateral, orientable surface without an edge. One cannot pass from the interior to the exterior without crossing an edge, a frontier, without creating a phenomenon of rupture. It is therefore mathematically impossible to inscribe a unilateral and non-orientable Moebius strip on its surface. To inscribe it, there must be a particular process with specific events, which we are going to trace: one first makes a double-buckle shaped cut, an interior eight which is also seen to be equivalent to the edge of a Moebius strip.³ Thanks to this process, one turns around the axial hole twice.

One then obtains a strip with four half-twists, even numbered, and thus a bilateral strip with two edges (materialized here by a different writing of two edges.)

With a slight transformation of the cut, we can establish a half-twist, often forgotten. The twist at the edge of the fold accounts for the axial hole in the torus.



In the first drawing, this half-twist is in fact reduced to a point. This reduction owes to our difficulty with the mental representation of the torus itself. Without this half-twist nothing any longer distinguishes the torus from the sphere. The half-twist is here the expression of the structure of the torus. However, the sticking together of these two half-twists one on top of the other allows us to create a Moebius strip. By these two half-twists, the torus "cut out" in this way is revealed to be the "combining as two thicknesses" (*revêtement à deux feuillets*) of the Moebius strip. The following drawings illustrate the process step by step.

Let us start again beginning with the cut:

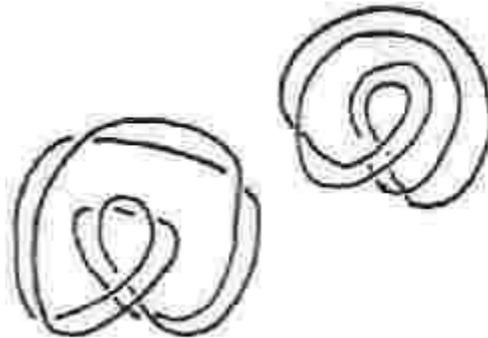
Because of its suppleness, we can reduce the surface of the torus to the space bordering the cut.

Two folded ends remain.

³ Cf. Chapter 2.

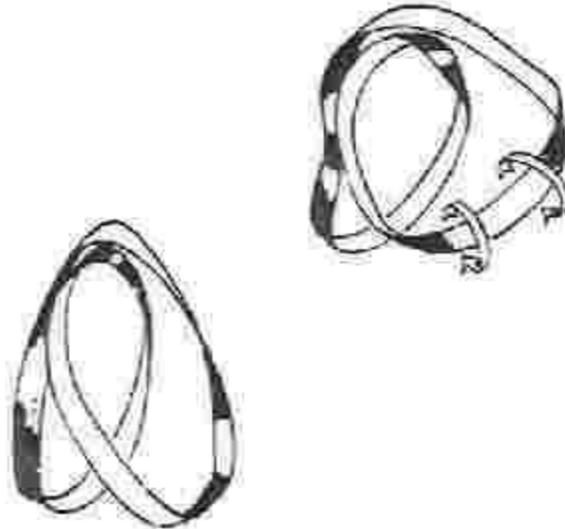


The surface crosses over itself.



We can then reduce the folds to the half-twist that they represent, then put the two buckles *en miroir*, and bring them together. Finally, we see how a sticking together of these two buckles, all the length of their surface, creates a unilateral Moebius strip with one half-twist.

This is what one calls the "combining as two thicknesses" of the Moebius strip.



Moreover, this trajectory is the reverse of the cutting of a Moebius strip. The latter creates a bilateral strip with four half-twists. At issue is a strip that can be cut out of a torus.

This combining as two thicknesses is the operation accounting for how the unconscious and consciousness "communicate by a toric world." This happens, for example, in what Freud calls the "double inscription": the same memory is inscribed in an unconscious and conscious chain. The Moebius strip gives us the structure of the signifying chain and the cutting effects of certain words, as interpretation.

This relationship of the Moebius strip with the structure of the torus links a whole series of questions to the moment of the first identifications and the learning of language (*la langue*). *A contrario*, psychosis and its possible possible psychoanalytic treatment perhaps finds a new direction thanks to these formulations. What, in effect, is interpretation in the psychotic structure? Is it not rather the matter of a construction?

We might ask ourselves if it is a matter of a cutting of the sticking together of a combination of two thicknesses? The structure itself of language (*la langue*) is laid out here (*est ici sur le métier*), and topological objects give us the means of formalizing it.