

Chapter 4: The Cross-Cap

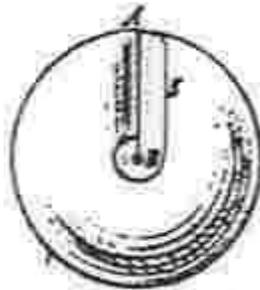
The drawing of the cross-cap, as we find it in the teachings of Lacan, dates from 1890. It appears for the first time in the work of Van Dyck.

This closed surface, without an edge and unilateral,* is an abstract object whose mathematical definition preceded its representation; it is a presentation of the projective plane.

The projective plane is the space in which projective geometry is conceived. It is defined by the adjunction of a point called "by convention" at infinity to the Cartesian plane (the three coordinates x, y, z , give us its calculation).

Toward the 1880's, mathematicians like Felix Klein and Schäfli recognized in this abstract object the quality of being unilateral, or non-orientable.*

Thanks to Van Dyck, we find the projective plane represented for the first time in the not very explicit drawing below:



Speaking rigorously, this drawing is an immersion* of the projective plane into our ordinary space. The projective plane itself must be conceived of as a space on the same basis as our ordinary space.

The surface is continuous and accounts for the infinity of the space of the projective plane (the term "continuous" means "without edge"). It is also unilateral, that is, its interior is continuous with the exterior. It is a closed surface, but one that does not delimit space.

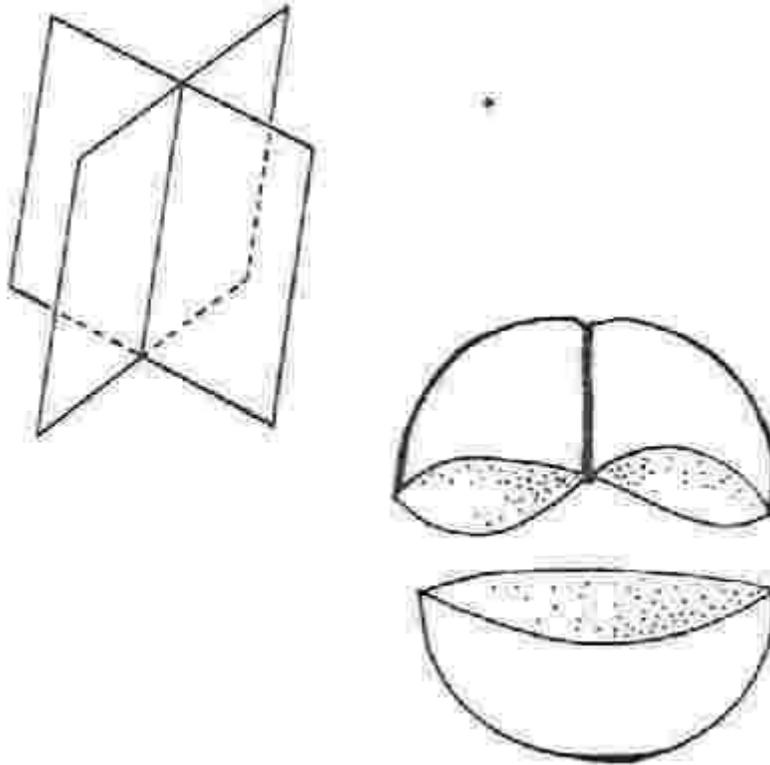
The lines of the drawings are not borders, but lines of curves and an intersection (the vertical line in the middle of the egg in the photo).

The intersection is a phenomenon that escapes our everyday, intuitive perception. Its difficulty arises from the fact that cross-cap as object is not physically realizable in our ordinary space except at the cost of this intersection. We must remember that there is nothing extraordinary about this, since it is a question of another space that should allow us to perceive the laws of our ordinary space *á contrario*. We submit to these laws as if they were self-evident. The cross-cap has the merit of bringing this evidence into question.

Thinking its structure requires an effort of the imagination that allows for particular phenomena like the intersection or "double-line"; two surfaces intersect, but it is a question of an abstract intersection that is situated nowhere. Lacan calls it a "pseudo-intersection."

Two surfaces intersect, passing through each other along an arbitrarily drawn line. If we were to represent a little ant walking on one of its surfaces, it would follow its trajectory without knowing that another surface had crossed the first.

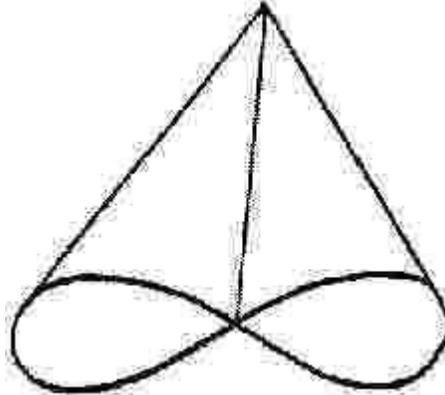
The cross-cap is a sphere creased by a line of intersection. On the one hand, it is closed like a sphere; the line is a line of the fold. However, at the top of the drawing, the cross-cap is closed in reversing the two thicknesses. They intersect along a double line.



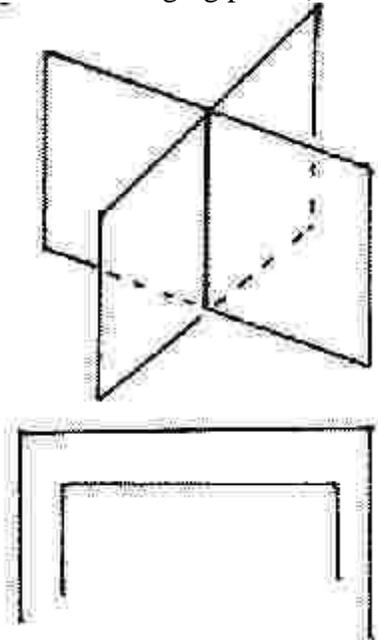
This structure possesses two points that are particularly difficult to think. The first is that where this double line stops. The surface re-closes itself at this point.



When one cuts away this particularity alone, one obtains what we call the eight-cone: a cone whose surface intersects itself. The line of the base of the cone describes a lengthened figure eight.



The second point, which puts our imaginary to the toughest test so far, is the point of departure of this line of intersection. It represents the point of the impossible to think, but not to write. It is the point off the line (*point hors ligne*), the point where one passes locally from a situation where two surfaces are posed one over the other to the situation where the same two surfaces intersect. One could say that this point sums up the set of the characteristics of the cross-cap, transforming a bilateral object (two thicknesses posed one over the other) into a unilateral object (these two thicknesses exchanging places).



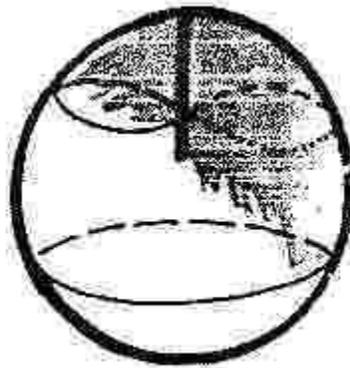
This intersection has as its consequence the putting in continuity of the external face with the internal face.

If we consider the cross-cap as pure surface without thickness, the interior of the sphere communicates with the exterior. In the same manner, the Moebius strip puts the topside in continuity with the underside.

We can now write the different trajectories possible for our ant: the four possibilities are presented as follows:

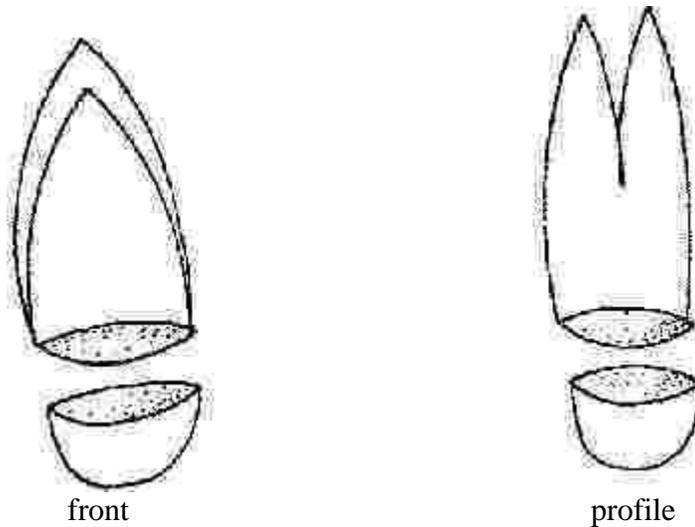
_____ topside in front

_____ underside behind
 topside in front
 - - - - - topside behind

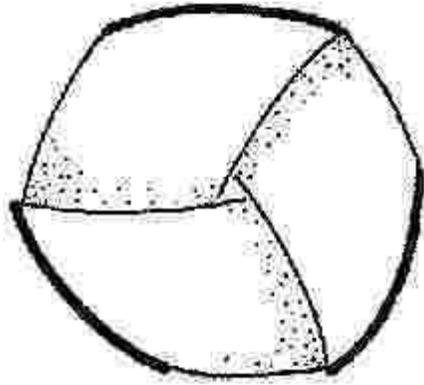


Lacan calls this figure a *bonnet croisé*, a translation of "cross-cap."

He also at times employs the term "bishop's miter." He then alludes to the real form of a bishop's miter creased toward the middle, a miter of which the bottom would be closed in a spherical manner, and of which the intersection would not be a banal effect of sewing, but effectively the passage from behind of what was in front and visa-versa.



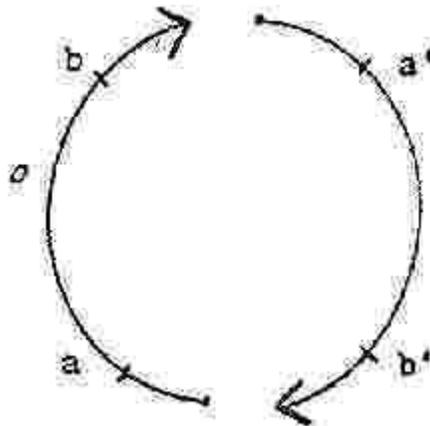
The Boy surface is another immersion of the projective plane, constructed on a Moebius strip. While the cross-cap is constructed from a Moebius strip with one half-twist, the Boy surface is constructed from a Moebius strip with three half-twists, as is shown in the following drawing: it is not a question of a demonstration, but only of a drawing that renders perceptible the construction of the Boy surface.



But let us return to the definition of the projective plane itself.

In mathematics, the projective plane is first of all a structure of organization such that each point of the sphere is associated with another diametrically opposed point.

Hence, a point situated at place (a) will correspond, will be identical to, a point situated at a place diametrically opposed to it (a'); it's the same for (b) and (b'). We in this way describe a circular relation, a structure where the trajectories entangle themselves in a star.

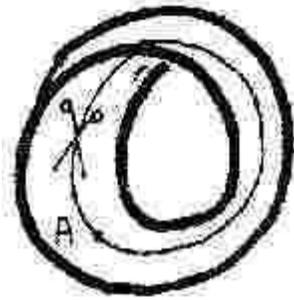


Since we are also dealing with a continuous structure, there is no possibility of marking our starting point.

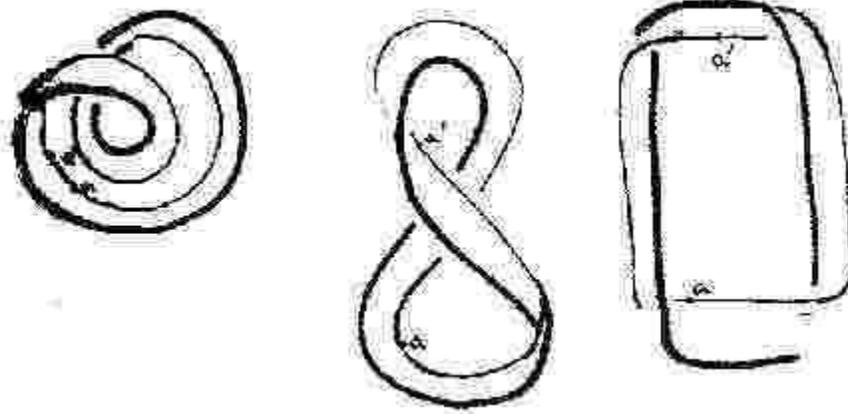
We can construct a Moebius strip beginning with this definition. We start by giving a simple strip with two faces on which we make correspond to one point a diametrically opposed point. We then effect a series of constructions that transform the simple strip into a Moebius strip.¹

The following drawings show this construction in reversing the trajectory. The reader will better follow the disappearance of the Moebius strip and the fate of the point marked A on the two edges of the cut.

¹ Cf. Chapter 2.



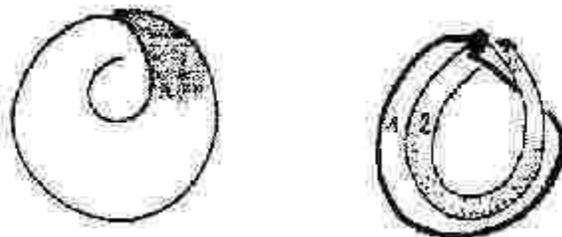
The problem of twists, of an even number, which maintain the two faces of the simple strip, will be dealt with in the next chapter. It is a question of a characteristic that does not modify the intrinsic nature of the surface.



On the cross-cap, this equivalence between diametrically opposed points creates an entanglement that still leaves an unimaginable point at the center. Lacan situates it thusly: "This circular relation must be perceived as a sort of rayed intersection concentrating the exchange of a point with the opposed point on the single edge of this hole, and concentrating it around a vast central intersection that escapes our thought."²

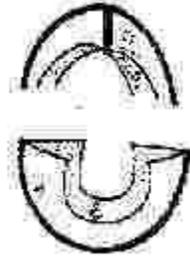
As we can guess from the preceding drawings, one can construct a cross-cap from a Moebius strip. *A contrario*, one can define a Moebius strip with a cross-cap; it is a holed cross-cap.

To obtain a cross-cap, we must first crease the Moebius strip lengthwise. Then, at the place of the twist, whether it is to the right or to the left, there is a problem.



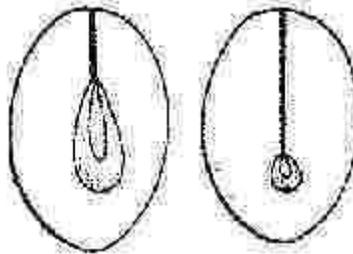
The two sides of the fold must cross each other (the dotted numbers on the drawing indicate their continuous movement onto the underside of the surface); the double line or intersection inscribes this structure of the crossing of surfaces:

² Seminar of June 6, 1962, unpublished, *L'Indentification*.



All that remains then is to close the whole (*l'ensemble*) of the surface, which is to reduce the hole to a point.

Before the closing of the hole, this object is a Moebius strip; after this same closing, it is a cross-cap. This is why one speaks of the cross-cap in reference to a Moebian space.



This point remains irreducible. As soon as it disappears as a hole, what becomes of the surfaces is a mystery, since nothing any longer makes a border.

This is what permits Lacan to turn the question inside out. The hole, this hole, becomes the construction's point of departure: the cross-cap is an organization of the hole. "It conjures away the hole."³

"It is a surface that in some way has taken the place of the hole, a surface where one divines it, although the important thing for the structure of the hole remains the central point, where the line of pseudo-intersection begins" (*L'Identification* 5/23/62).

This manner of making the cross-cap appear gives to its surface an altogether particular dimension. This central point, in fact, if we can subsume all the history of mathematics under the "point at infinity," is also what permits Lacan to make a new usage of this topological object. He is interested in the cut and the place of this point-hole in the effects of the cut.

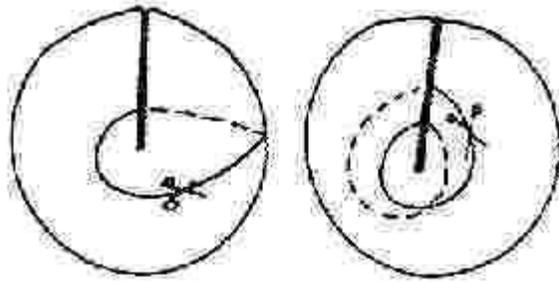
This usage of the cut completely subverts mathematical discourse. The cut is an operation. One can never say enough about the originality of this step.

This operative quality of the cut leads us understand Lacan's interest in the Borromean knot (whose definition is supported by this operation of cutting). But we will come back to that. We must first present in detail this cut on the cross-cap, whose characteristics permit an economic and synthetic formulation of the analytic experience.

The cut has a particular relation with the central point of the cross-cap, as can be shown by this series of drawings: a simple cut opens the surface without separating it.

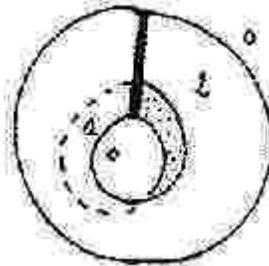
But it suffices to make this cut turn around the central hole to divide the surface:

³ Seminar of May 23, 1962, unpublished, *L'Identification*.



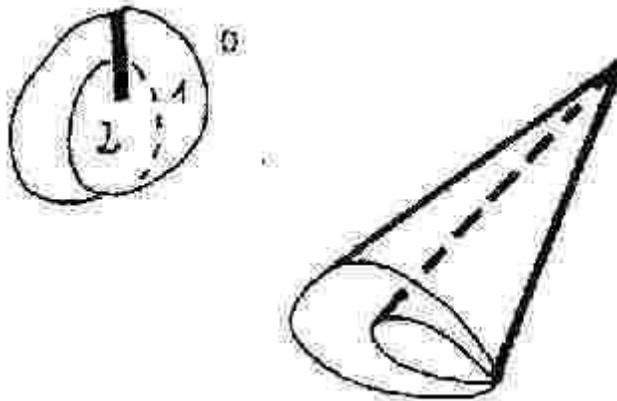
One then obtains two pieces:

--a Moebius strip that can be either left-handed or right-handed when it is unfolded (it is the strip folded lengthwise).



--a disk, which possesses this particular point of the cross-cap. (The numbers refer to the number of thicknesses).

It is a question of a bilateral disk, with a double line, as is shown in the following drawing: it seems also to be related to an eight-cone, constructed on an interior eight.



The bilateral disk, with two distinct faces, bears the point essential to the cross-cap's surface. The double quality of this object, by definition bilateral and with two faces, but nonetheless bearing the point essential to the unilateral structure of the cross-cap, gives a particular status to this disk.

Intuitively, one indeed perceives that it suffices, starting with the disk, to reclose the interior eight on itself to once more obtain a cross-cap. It is this disk that Lacan identifies with the object (a):

"It is in articulating the function of this point that we can find all kinds of felicitous formulas that permit us to conceive of the function of the phallus at the center of the constitution of the object of desire" (*L'Identification* 6/27/62).

The point of the impossible to think, but not to write, the point-off-the-line (*point hors ligne*), is identified by Lacan with the Φ .

For Lacan, the cross-cap is the "topological support that we can give to the fantasy" (6/27/62).

The fantasy is a cutting of the cross-cap that detaches an object without specular image, the central disk, from a Moebius strip.

To be more rigorous, we should add that this cut created the Moebius strip and the disk bearing the central point.

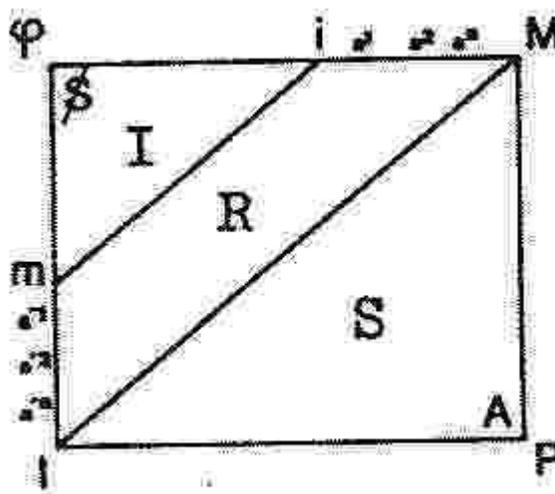
The usage Lacan makes of this cross-cap figure is original; the cut is an operation that does not function to underscore a definition but to provoke a transformation formalized as such: at issue is the constitution of the fantasy. The formulation of the fantasy is written with this remainder, the object (a), detached from a Moebius strip, which represents the barred subject on the basis of this loss. We see how the operation of the cut synthesizes, sums up, the definition of the subject in relation to the object, at the same time as their relations. The latter are formalized elsewhere as "separation-alienation," written with the matheme: $\$ \diamond a$.

We can now refer to the Schema R, drawn by Lacan several years before he defined it in a note of 1967 as the laying out of a projective plane:

"Perhaps it will be of interest to recognize that enigmatically, but perfectly readably for those who know what follows, as is the case for those who claim to be supported by it, that what the schema R lays out is the projective plane."

"Specifically, the points of which it is not by chance (nor for fun) that we have chosen the letters to which they correspond mMI and which are those that give the framework for the only worthwhile cut in this schema (the cut miMI) well enough indicating that this cut isolates in the field a Moebius strip."⁴

The Schema R is therefore a putting flat of the projective plane or the laying out of the cross-cap in which two cuts are made: one is situated at the place of the line of intersection in the classic drawing. This cut permits the delimitation of a surface where four points are establishable and can then be assimilated to a square. On this square, the cut of the fantasy follows the lines delimiting "the field of reality," marked R on the Schema.



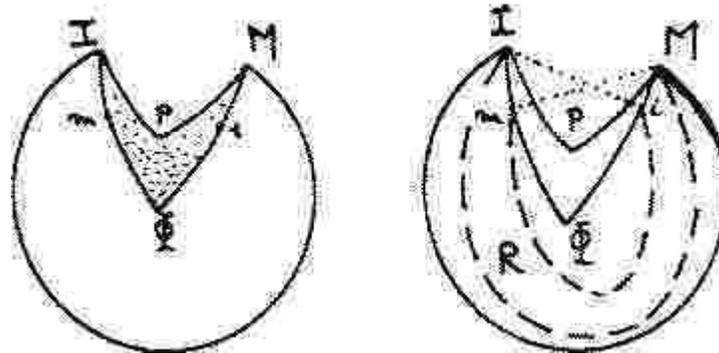
⁴ "On the Possible Treatment of Psychosis," *Écrits*

The following drawings will show these two cuts and their outcome on the cross-cap.
 The square of Schema R takes the form of a pouch, of a sphere, from which one withdraws a portion.

One can then write on the surface the letters of the Schema R.

The cross in dotted lines evokes as much the re-closing of the cross-cap as the twist in the Moebius strip, miMI.

(Moreover, on the trajectory of the cut called "of the fantasy," there is a displacement of the above-beneath.)



Based on this, the field of reality in the Schema R is directly readable on the cross-cap, as the cut in the form of an interior eight. In question is an operation that effects the fundamental fantasy: at once separating and joining a subject and an object. At the same time, the fantasy constitutes the framework of our perception of reality.

We here see again the utilization of the Moebius strip as an operation of the cut that permits us to unveil structure. It renders possible a laying out that belongs only to speech. We see how topology synthesizes, sums up, diverse aspects while taking them up in a same synchronic, structural, access to meaning. It thereby permits us to bring to light the operation, the functioning, of concepts between themselves, and it is in this that it is structure.

Fundamentally, it is a question of a taking into account of the operative effects of speech as interpretation.

The transference is then the space in which this cut operates. (Let us say, rather, that one must add a conception of the transference.) It is also on the figure of the cross-cap that Lacan establishes the structure of the transference. The analytic situation is thus articulated by the same object in the measure where its operation brings to light the fundamental fantasy.

In seminar XI, Lacan speaks from the place of the analyst in the transference,⁵ and says how he situates it in the central place of the construction.

To support his development, Lacan takes up again his celebrated formula dating from ten years earlier: "the unconscious is structured like a language." He poses the unconscious as an effect of speech on the subject. It is then that is posed the question of situating in this formulation the Freudian discovery of the importance of sexuality. Lacan makes use of the Moebius strip and, hence, of the cross-cap, to knot these two aspects of the unconscious, that which Freud brings to light in his study of "jokes" (*les mots d'esprit*) and that of the repression analyzed in his "Studies on Hysteria."

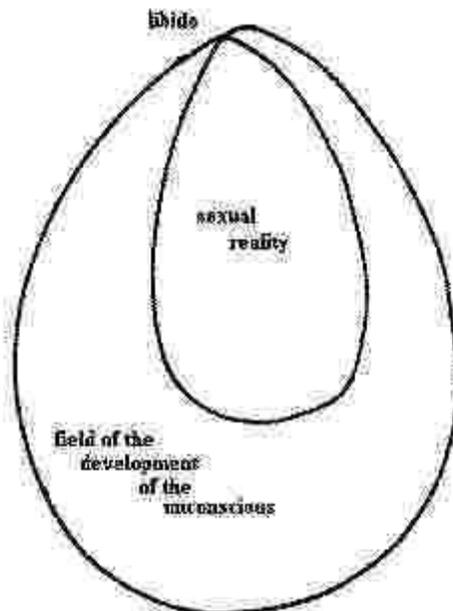
To show the importance of this question of the articulation of discourse to sex, Lacan begins by supporting his views *a contrario* on Jung's reflections. He shows how, to effect this knotting, which Freud leaves as is, Jung comes to consider unconscious sexuality as a

⁵ Séminar XI, *Quatre concepts fondamentaux de la psychanalyse* Le

remanaging of archaic thought. (As has since been shown by Levi-Strauss's discoveries, it seems that the signifier came into the world beginning with the sexual difference.) In this way, Jung founds the notion of the archetype and then sacrifices to the recognition of the original, sexuality as material of the unconscious. If sexuality is present in the unconscious, it is in the name of its original importance in the constitution of language.

We know that Freud always refused this consequence, holding as essential the sexual reality of the libido. We should remember that he went so far as to break with Jung, long considered his successor.

Lacan takes this articulation of sex to speech up again and makes use of the Moebius strip and of the space that it finds as a point of departure for making perceived the points of conjunction and disjunction of the one with the other. He thus makes a first conceptual distinction: he speaks of a field of unconscious development to refer to its language aspect, distinguishing it from the reality of the unconscious, which is sexual. The Libido is point of crossing that can be drawn on the surfaces enclosed by an interior eight: it is a question of the point of junction between the two fields.

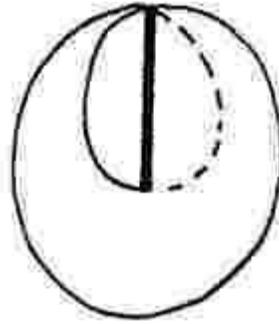


For the desiring subject, it is beginning at the point where he desires that the connotation of reality is given to these perceptions. Hallucination shows it *a contrario*.

Due to the defiles of the signifier and their discontinuous nature, the demand articulated in signifiers always leaves a remainder. Desire is metonymic and runs beneath the chain. However, it is only it that insures the cohesion of the discontinuous elements that are the words. It is for a desiring subject that the sentence closes itself on a sense.

Lacan adds that this point of junction "libido" is inscribed in the transference: one must beware of conceiving of this formulation as subjectivized, representing an individual in his functioning.

At issue is the analytic situation. For this drawing in which one lobe hides another, depicting the Moebius strip, is here taken entirely in a surface. What the putting in place of the transference effects is the crossing of this hidden lobe to the front, along a line of intersection.



It is a question already of the cross-cap, the structure of which we recognize.

Lacan gives this description of the operation:

"You can obtain the cross-cap beginning with the interior eight. Unite two-by-two the edges as they are presented here, with a complementary surface, and close it. Somehow, it plays the same complementary role in relation to the initial eight as a sphere does in relation to a circle, a sphere that would enclose what the circle already would offer to contain. And indeed this surface is a Moebian surface, its topside continuing its underside. There is a second necessity that arises from this figure: to close its curve it must somewhere traverse the preceding surface, at this point here, along the line that I reproduce."

Lacan adds: "This image allows us to figure desire as place of a junction of the field of demand where are presented the syncopes of the unconscious with sexual reality."

"All of that depends on a line we will call desire, tied to demand."

"What is this desire? Do you think it is there that I designate the incidence of the transference. Yes and no. You will see that the thing does not go without saying, if I tell you that the desire in question is the desire of the analyst."

For the unconscious to unveil itself, for interpretation to operate with its effects of alleviating psychic suffering, sedating conflicts, for the cut of speech to operate, the analytic situation in its entirety is required, and precisely the desire of the analyst, which founds the transference.

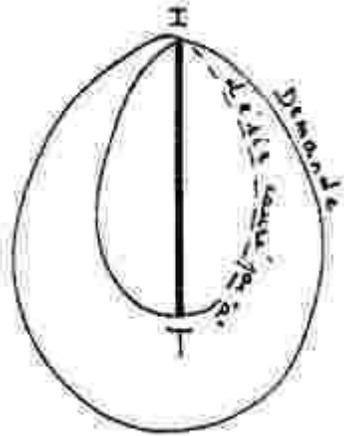
If this desire is noted at the central point of the cross-cap, it is because represents the analyst as object (a), marked by the cut from the object and its loss.

The transference is founded on the desire of the analyst, on the cut that, for him, has signaled the advent of desire and the detachment of the object.

The cross-cap is introduced as a space made necessary by the formulation of the cure. It is important to take the time to establish this passage from the topology of illustration to the equivalence between space and structure.

For that, it helps to clarify the formulation of the transference.

Lacan writes on the interior eight the places of the different notions we have taken up.



He situates the analyst on the line between (I), the point of fascinating identification, and (a), the object, at the point of departure of the line, where he writes (T), basal point of the transference. He then insists on this beyond of identification, of which we must remember that it signals, for analysts on the other side of the Atlantic, the end of analysis. This beyond is defined by the relation to and the distance of the object (a) from the big idealizing (I) of identification.

We again find the letters of the Schema R and the contour of the cut the fantasy puts in operation. To let appear in the cure the place of the object is also to permit the fantasy and the cut that it puts into operation between the subject and the object to be put in place. It is by this operation that the subject can identify with what he desires.

We thus see how the cross-cap is a structure fundamental to the formulation of analysis for Lacan, above all because it allows the showing of the operation effected by speech. It gives support to the analytic act by marking the conceptual points between which it operates.

What does a psychoanalyst do? Lacan gives a first answer here.

Later, with the Borromean knot, he will give another theory of this set. But one can already remark that he then situates the cut at a place still more central to the constitution itself of the concepts in play.