Chapter 6: From Surfaces to Knots

In his topological advances, Lacan moved from a usage of surface structures to knots.

This movement was problematic to the extent that there is no mathematical conception that englobes these two parts, although they are very much linked in general topology.

Thus the edge of a Moebius strip with three half-twists is seen to be knotted in a trefle-knot: on a manipulation (*pliage*) of this knot is constructed another immersion of the projective plane known as the "surface of Boy,"[1] the structure of which is shown in the following drawings:
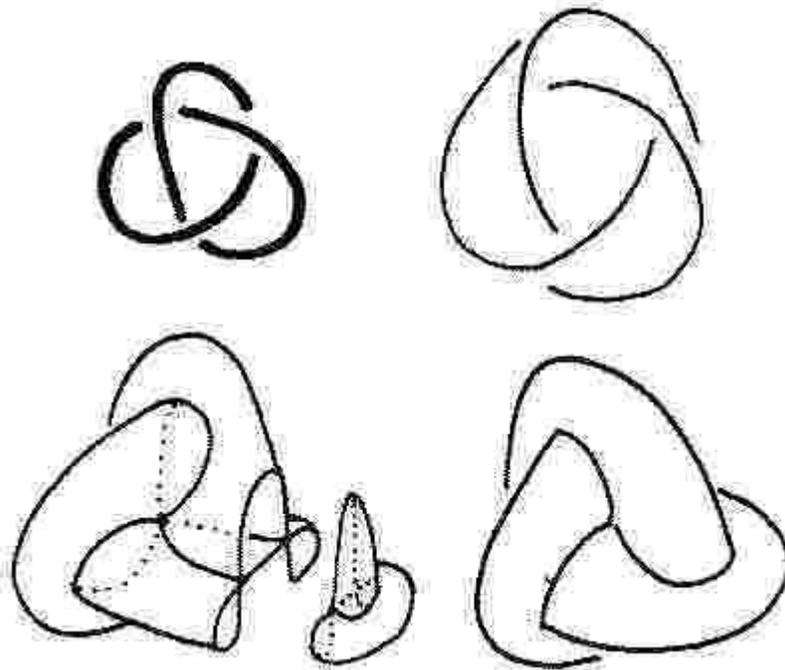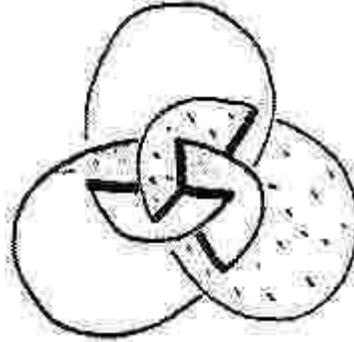

This surface of Boy puts in place what is called a "triple point." When three surfaces intersect, they define a point: when there are two, they define a line (as we have seen).
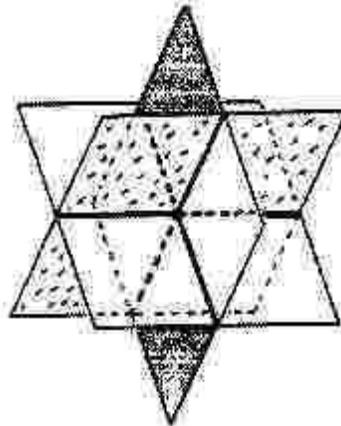
This triple point also appears in a mounting of surfaces on the Borromean knot:

---

[1] Drawing of J.C. Petit, published in *Pour la Science*, January 1981.

How are the knots and the surfaces linked?

Several responses, or rather several trajectories, allow us to account for this liaison and for Lacan's progress from one to the other.

The surfaces and knots treat of some possible articulations between the elements of a structure.

The surfaces with their spatial representation put in question, even to work, the schematic conception of structure, which is already at work in the labors of Levi-Strauss. The notion of space is fundamental.

Knots, on the other hand, are supported exclusively by the operation of the cut.

On the surfaces, Lacan makes an operative use of the cut. In this respect, he does something new. This usage is essential to how Lacanian topology serves psychoanalysis. The cut accounts for the analytic act: essentially, for interpretation.

We must now insist on the subversive aspect of this usage of the cut on surfaces.

In mathematics, the cut serves some definitions of surfaces; care is taken to differentiate them so as to classify them and number them (cf. genre*).

Lacan completely subverts this logic. He draws on the wish to work with the surface, to produce a history thereof, to cross over it in a movement, to permit speech, because it emerges from the a-temporal laying out of structures.

This subversion leads Lacan to knot theory. Knot theory puts this operation to work. A knot is defined negatively by the necessity of the cut:

A knot is all the interlacings of threads that must be cut to make it disappear. The cut also leads to knots topologically. It is cuts on surfaces that create knots.

The result of the cutting operation is a knot:

Hence, a median cut on the Moebius strip leaves the strip whole but with four half-twists.

This strip cut again separates into two pieces, albeit knotted. The photographs make this quite clear.

On the Moebian strip, a cut that encircles the surface twice separates the surface into two pieces, but they are knotted.
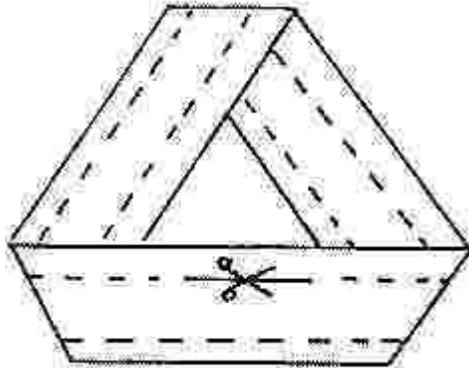
Of these two pieces, one is a Moebius strip, with one half-twist, as at first, and the other is a strip with four half twists. We create two objects, one bilateral, the other unilateral.
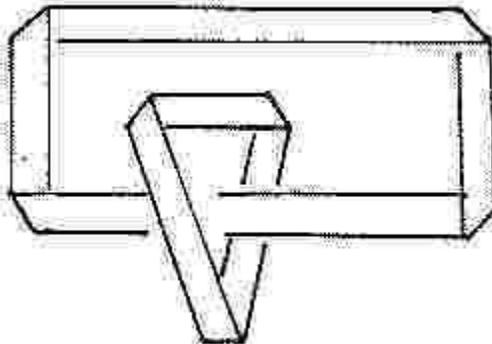
This operation evokes the so-called cutting "of the fantasy" produced (*opérée*) on the cross-cap.[2] On the other hand, the knotting evokes the so-called knot of the fantasy: there is an interior eight and a round: an edge of a Moebius strip and an edge of a disk (see photo).
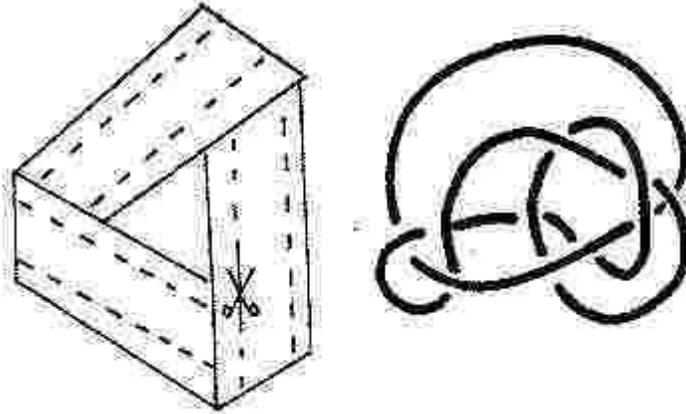
---

[2] Cutting of the cross-cap, cf. Chapter 5.
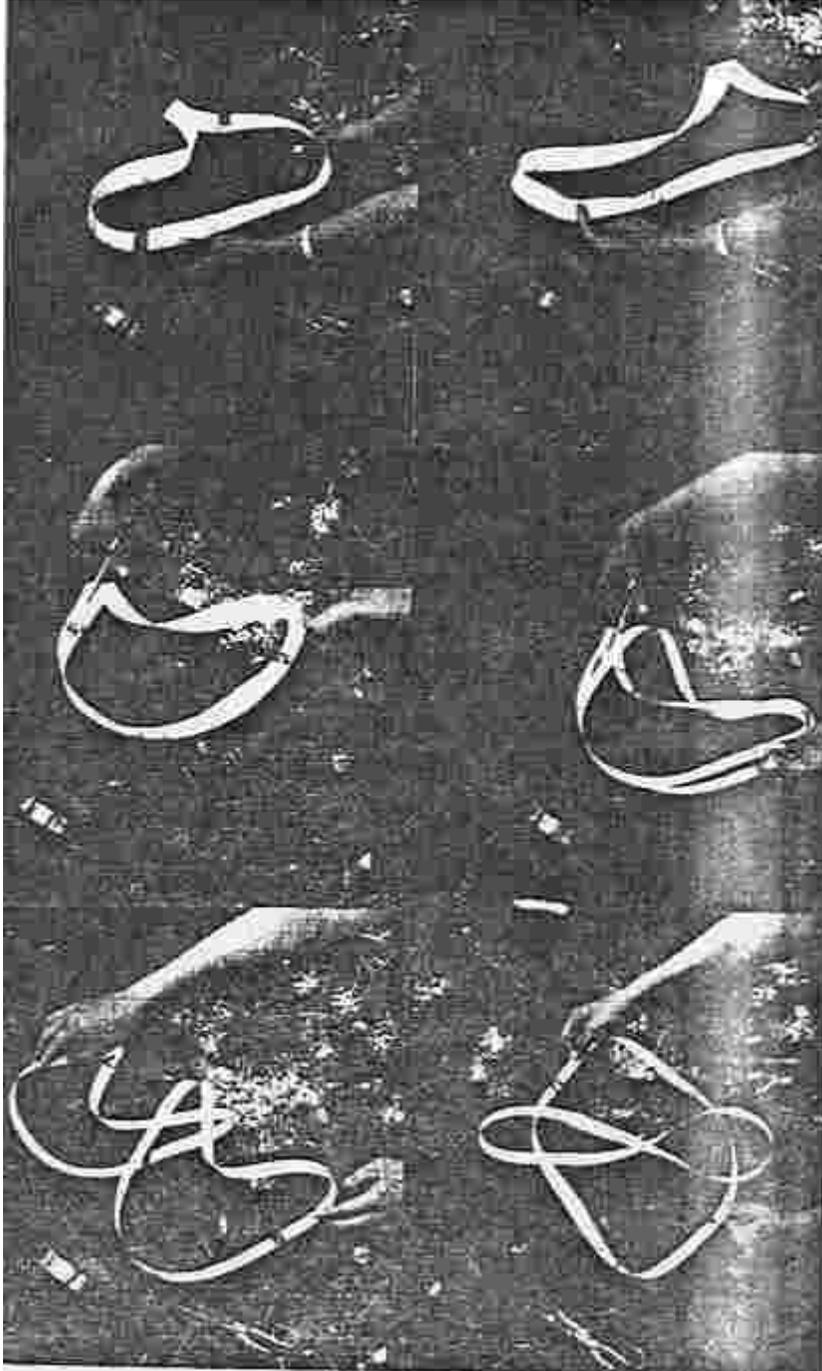
Cutting of a strip with four half-twists.

In other words, the fantasy as a cut on the surfaces, or knot, is at the heart of this passage from surfaces to knots.

On the Moebius strip with three half-twists, an interior-eight-shaped cut produces two pieces knotted like this:

There are two above-beneaths too many for the knot of the fantasy. What is surprising is that the real of our everyday space is maintained despite the impossibility of arriving, by a cutting of a surface, at the so-called knot of the fantasy.

Let us take up again the problem beginning with the twist in surfaces that are the expression of our everyday space and of the characteristics proper to it. We have seen from a variety of perspectives the importance of the twist in the topology of surfaces and how they allow us to account for the analytic experience. The twist plays a primordial role here, since the constitution of the object (a) and the misrecognition belonging to the constitution of the Ego is brought into play around the difference between the specular and the non-specular. The twist retains its fundamental place in this distortion.

However, the topology of mathematicians up to now evacuates this twist. A strip is bilateral whatever the number of its half-twists, provided that this number is even. In the field of psychoanalysis the twist of surface renders the cut operatory and creative of knots. Based on this, the half-twist of a Moebius strip is exemplary:
It writes the fundamental difference of above-beneath.

There are two of them, just as there are two Moebius strips, depending on the directions of the twist:

The drawing of the interior eight accounting for them brings in an above-beneath and a beneath-above. The Moebius strip is here once again exemplary.
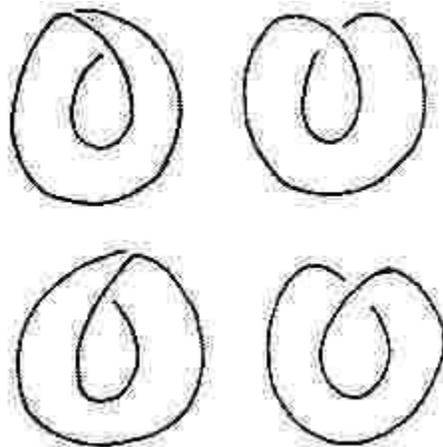
Moreover, although the above-beneath writes the direction of the twist as right or left, it the also dispenses with the concave or convex characteristic of a surface. In this sense, it (and the knot) proceeds from a phenomenon of the simplification of surfaces.

Thus, when we retrace the history of the cutting of a Moebius strip with one half-twist, we see how this history, centered on the twists, is subject to complications.
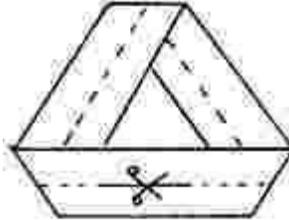
The cut creates a second edge and multiplies the half-twists by two; we remove the forward buckle (*boucle de devant*), which makes one half-twist disappear from one of the branches of the eight, and leaves two on the other: the half-twists are displaced.
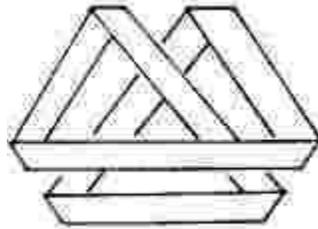
We suppress the self-crossing in unfolding the lower buckle: two half-twists disappear but we cannot truly flatten the unfolded branch.
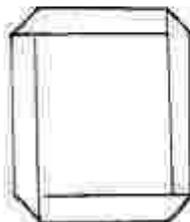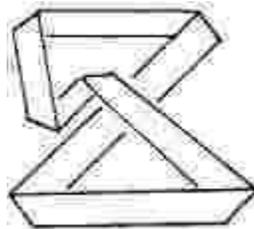
The self-crossing, "the above-beneath," is equivalent to two half-twists. We displace some of the half-twists and we transform one twist, which can be called convex, into a concave twist, which changes the direction of the twist.

Here we see four events susceptible of affecting a half-twist: right, left; before, behind. The above-beneath lets one of these oppositions fall:

At this stage, if we think of it terms of a movement along threads, there are only two possible outcomes: the passing of one thread under another--which can be the same at another moment of its movement (self-crossing)--or above it, while a surface twists to the left, to the right, before or behind.

Based on this, the history of the same cut on a Moebius strip can be retraced more simply.

We only retain the history of its edges and their knotting:

we unfold the buckle

We suppress the
above-beneaths of the two
buckles (they are not knotted),

then the
self-crossing

note here that this
knot is not the
knot of the
fantasy, because
the interior buckle
of the eight does
not hold.

Thus, if the knot is a simplification of writing, for the fantasy the two writings have their *raison d'être*. In fact, it is a symbolic operation and an imaginary function.

The cutting of the cross-cap gives us a Moebius strip the direction whose twist is not specified.

In the place of the twist, there is a line of intersection.

Here, the surface gives rise to a difference without effect.

On the other hand, the disk left by the cut keeps its pertinence, in relation to the simple round of the knot, since it is constructed on an interior eight.

Let us remember that it bears the remarkable point Φ:
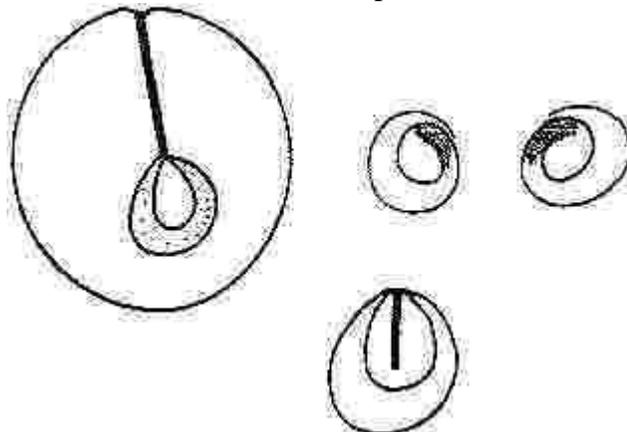
The so-called "knot of the fantasy" is also formed by two threads, a disk-shaped simple round and the interior eight of a Moebius strip. Their places are interchangeable:

The one becomes the other, and visa-versa.

The exchange writes, at a level internal to the knots, the effects of the mirror on twisted surfaces. The fantasy is an imaginary function.
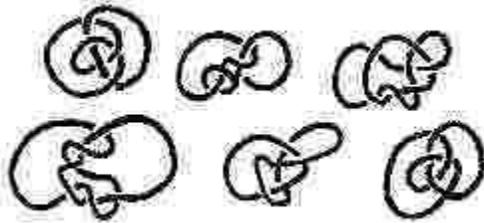
We see how this knot is situated on the cross-cap, as a cut before the twist, an effect of the symbolic constitution of the object (a) and with the knots that englobe specular effects themselves. It is in fact a knot whose multiple forms can offer a near symmetry.[3] Moreover, "there are" two, a left-handed and right-handed one, we will say, although it is no longer here a question of a half-twist: the one is the specular image of the other.
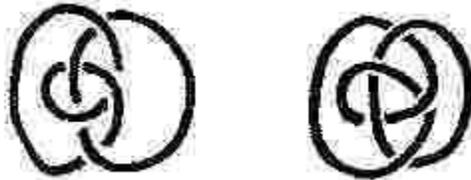
Thus, from the twist in surfaces to knots there is a writing, a simplification, but also a radical change in nature, a leap from the imaginary to the symbolic that cannot be evacuated.

If the cut creates knots on the surface, they are knots formed from two threads. They are all Borromean. The Borromean quality is trivial to them. Whatever the knotting, since there are only, two consistencies, it suffices to cut a single thread for the knot to disappear. The Borromean quality begins at three.

The negative function of the cut on the knots contributes to the definition as an operation *a contrario*. "A knot is an interlacing (*enrelacs*) that a cut can make disappear."

The Borromean knot plays an exemplary role here in that it is the effect of the most simple knotting. One cut and, whatever the number of threads in play, the knot disappears.

A contrario, it is necessary to speak of surfaces to bring the cut into operation, before three consistencies.

The omnipresence of the number three in this passage owes to structure. The symbolic makes three, allows for counting and naming. Before it, as we have seen, there is a slippage, a constant exchange between the one and the other.

We can now, *apropos* of this topological work, speak of an effect that gives all of its value to the *Lacanian* advance.

In *Scilicet* n. 2/3, the article titled "For a Logic of the Fantasy" develops an effect of exclusion between "topos and logos." The articulation of the one to the other is impossible even though it is real and, moreover, it presents us with the necessity of a writing (as third term: you read this work; you do not hear it).

In fact, if "logos" refers (*renvoie*) to speech, "topos" is nothing other than the place of the body. The subject is the effect of this exclusion.

Why, then, does the place of the body find itself excluded from speech? We say to begin with that this exclusion owes to the fact of its being not-two. Let us begin (*Prenons acte*) by

---

[3] Cf. the book of Jean-François Chabaud: *Le noeud dit du fantasme.*

noting that the unity of the body is opposed to the system of logic, which is defined starting with the dyad, under the rubric of the principal of identity, of non-contradiction or of bivalence. There has to be a proposition "x f of y."

The place of the body can only be translated (*se traduire*) into the order of language by a series of disjunctions from which the unconscious acts: "the field of formations of the unconscious with which the psychoanalyst is concerned is, however, that of the compromise formations that, in the mode of denial (*dénégation*), allow it to retranslate place into logic," into a series of divisions:

the division man-woman,

that as object of desire (a) and the set of the effects of Demand (A),

that of the body and jouissance, the division between knowledge and truth. In this way of seeing things, working with surfaces is a challenge for speech. It is situated in the order of the body, of space, and not in that of the metonymy proper to speech, to the "signifying chain."

The concepts are no longer points of reference; they are shown in a simultaneous vision, a space proper where our corporal perceptive possibilities contribute. The drawing puts to work the scopic space in its relation with the imaginary of representation. Manipulation brings into play the skill of the hands, what is concrete in the movements of the body.

In his final seminars, Lacan no longer spoke of a topology. The position of the work in this field of place and its characteristics entails the loss of the possibility of knotting a discursive thread.

The real of the drawing and of its effects of representation that unfold there take their place in a laying out of the atemporal structure of words. The beginning and the end of a sentence are not present. All of the threads are permitted, and all are particular, individual, subjective.

In the same way seeing things is situated Jacques-Alain Miller's warning in his "Commented Table of Graphic Representations" adjoined to Lacan's *Écrits*.[4] He expresses himself thusly: "If it is true that perception eclipses structure, a schema will infallibly lead the subject to forget in an intuitive image the analysis that supports it."

"It is the role of symbolism to interdict the imaginary capture."

Although the drawings are "graphic representations" of an analysis, they stage this imaginary capture. In Lacan's work, it is the operation of the cut that truly transforms these objects represented into a topology of the subject.

There is no longer an occultation of the symbolic in topology because this presence of an operation evokes, properly speaking at the interior of geometrical intuition, the furrowing of speech.

Elsewhere, at the end of this warning, Jacques-Alain Miller refers us "for learning the rules of the transformation of geometrical intuition into a topology of the subject" to the note, added in 1966, to the Schema R. We have seen how this note puts in place on the Schema R the cut essential to the structure of the cross-cap.

A contrario here we have what gives topology all of its worth in the approach to the psychoses. The limit-effects our perception encounters are to the image of the world of the psychotic. For a long time already, psychoanalysts of children have put drawing to work in their sessions. The topological notions of edge, of interior-exterior, of cut, of envelop, are constantly present there, as well as spaces where these notions are not differentiated.

---

[4] Jacques-Alain Miller, postface to the *Écrits* of Jacques Lacan.

We have shown by the intermediary of the cut that structure itself induces this passage from the writing of surfaces to knots, when one passes from two to three dimensions.

In a still more general way, we can mark this passage in the work of Lacan as he himself defined it. In his famous trilogy "real, symbolic, and imaginary," he recognizes that he began with the imaginary in the optical schema, then he took up the symbolic, and finally the real. The knot is real, and the real only appears as a third dimension. From the moment when three rounds of thread are in play, there are no longer specular effects. There is only a Borromean knot.

In this domain the cut is already a symbolic effect as operation, because it brings into play, by its effectuation itself, the dimension of language (*la langue*). The cut that counts, psychoanalytically speaking, is the interior eight, the edge of the Moebius strip, whose relationship with the creation of sense in language we have seen.

The omnipresence of the number three owes to structure. At issue is nothing other than the Oedipus complex. The Borromean knot articulates the place given its mythical version in the Oedipus complex; it clarifies this three, "papa, mama, and me," which functions from the beginning.

In relation to this beginning, the psychotic does not find his place and the neurotic only finds it at the cost of the imaginary castration from which he suffers. It is to the extent that this castration becomes symbolic that the neurotic finds a path toward jouissance.